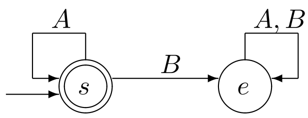
Solutions to Homework 3

Task: Apply the general algorithm for transforming the finite automaton into a regular language (i.e., a language described by a regular expression) to the following automaton.

This automaton has two states: s (= straight-A student) and e (= everyone else); s is the starting state, it is also the final state. The only two symbols are A and B.

- From s, A leads to s, and B to e.
- From e, any symbol leads back to e.

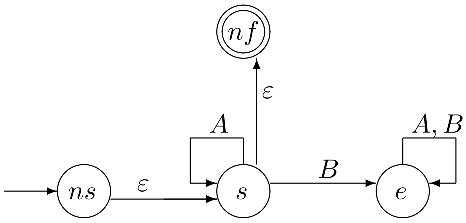
Solution. We start with the described automaton:



According to the general algorithm, first we add a new start state ns and a few final state f, and we add jumps:

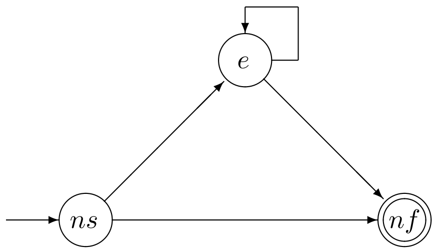
- from the old start state to the new start state, and
- from each old final state to the new final state.

As a result, we get the following automaton.



Then, we need to eliminate the two intermediate states s and e one by one. We can start with eliminating s or with eliminating e. Let us show what happens in both cases.

First version, when we first eliminate the state s. First, we draw all possible arrows:



Now, to find expressions to place at all these arrows, we will use the general formula

$$R'_{i,j} = R_{i,j} \cup (R_{i,k} R^*_{k,k} R_{k,j}),$$

where k is the state that we are eliminating, i.e., in this case, the state $k = s$.

By applying this formula, and by using simplification formulas described in the lecture, we get the following results:

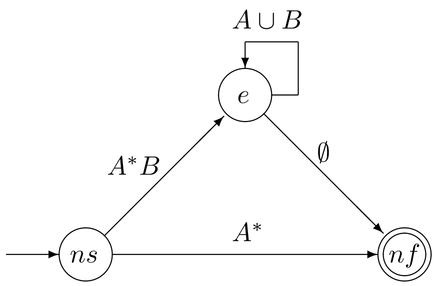
$$R'_{ns,e} = R_{ns,e} \cup (R_{ns,s} R^*_{s,s} R_{s,e}) = \emptyset \cup (\Lambda A^* B) = \emptyset \cup A^* B = A^* B;$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,s} R^*_{s,s} R_{s,nf}) = \emptyset \cup (\Lambda A^* \Lambda) = \emptyset \cup A^* = A^*;$$

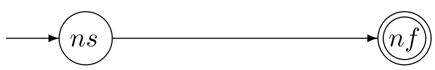
$$R'_{e,e} = R_{e,e} \cup (R_{e,s} R^*_{s,s} R_{s,e}) = (A \cup B) \cup (\emptyset \ldots) = (A \cup B) \cup \emptyset = A \cup B;$$

$$R'_{e,nf} = R_{e,nf} \cup (R_{e,s} R^*_{s,s} R_{s,nf}) = \emptyset \cup (\emptyset \ldots) = \emptyset \cup \emptyset = \emptyset.$$

Thus, the 3-state a-automaton takes the following form:



Now, all that remains to do is to go from here to the 2-state a-automaton by eliminating the remaining state e:



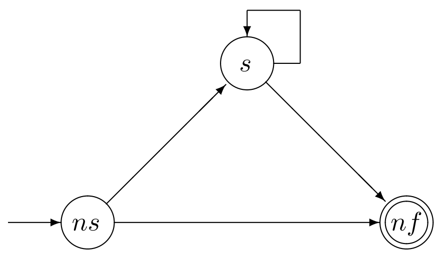
The final expression is the corresponding expression for $R'_{ns,nf}$:

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,e}R^*_{e,e}R_{e,nf}) =$$
$$A^* \cup (A^*B(A \cup B)^*\emptyset) = A^* \cup \emptyset = A^*.$$

The formula on the previous line is a regular expression corresponding to the original automaton.

First version – answer: A^*.

Second version, when we first eliminate the state e**.** First, we draw all possible arrows:



Now, to find expressions to place at all these arrows, we will use the general formula

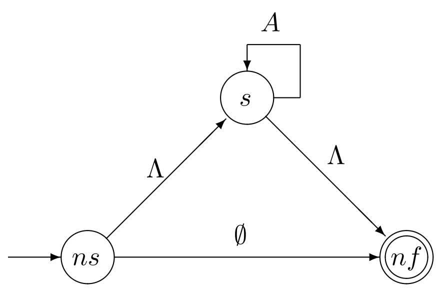
$$R'_{i,j} = R_{i,j} \cup (R_{i,k}R^*_{k,k}R_{k,j}),$$

where k is the state that we are eliminating, i.e., in this case, the state $k = e$.

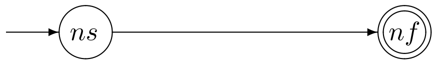
By applying this formula, and by using simplification formulas described in the lecture, we get the following results:

$$R'_{ns,s} = R_{ns,s} \cup (R_{ns,e}R^*_{e,e}R_{e,s}) = \Lambda \cup (\emptyset \ldots) = \Lambda \cup \emptyset = \Lambda;$$
$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,e}R^*_{e,e}R_{e,nf}) = \emptyset \cup (\emptyset \ldots) = \emptyset \cup \emptyset = \emptyset;$$
$$R'_{s,s} = R_{s,s} \cup (R_{s,e}R^*_{e,e}R_{e,s}) = A \cup (B(A \cup B)^*\emptyset) = A \cup \emptyset = A;$$
$$R'_{s,nf} = R_{s,nf} \cup (R_{s,e}R^*_{e,e}R_{e,nf}) = \Lambda \cup (B(A \cup B)^*\emptyset) = \Lambda \cup \emptyset = \Lambda.$$

Thus, the 3-state a-automaton takes the following form:



Now, all that remains to do is to go from here to the 2-state a-automaton by eliminating the remaining state s:



The final expression is the corresponding expression for $R'_{ns,nf}$:

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,s} R^*_{s,s} R_{s,nf}) =$$

$$\emptyset \cup (\Lambda A^* \Lambda) = A^*.$$

The formula A^* is also a regular expression corresponding to the original automaton.

Second version – answer: A^*.