

Solution to Problem 6

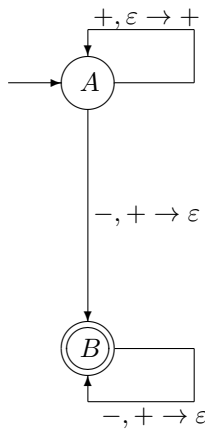
Task: Show, step by step, how the following pushdown automaton will recognize a sequence $++--$. This pushdown automaton has two states:

- the starting state A , and
- the final state B .

The transitions are as follows:

- From A to A , the transition is $+, \varepsilon \rightarrow +$;
- From A to B , the transition is: $-, + \rightarrow \varepsilon$;
- From B to B , the transition is $-, + \rightarrow \varepsilon$.

Solution. Our pushdown automaton has the following form:

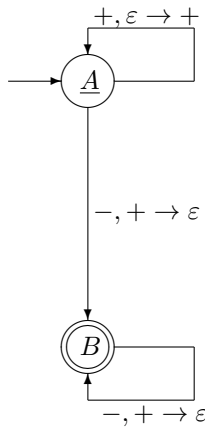


The main idea is that:

- first, we are in the state A ; we see $+$'s, and we push them into the stack;

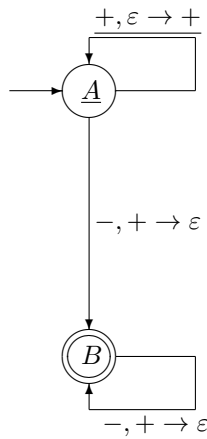
- then we see the first minus, we pop a plus and go to the second state B ;
- in this state B , every time we see a minus, we pop one plus from the stack.

So, we start in the starting state A with an empty stack.

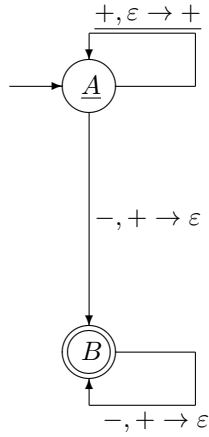


Then:

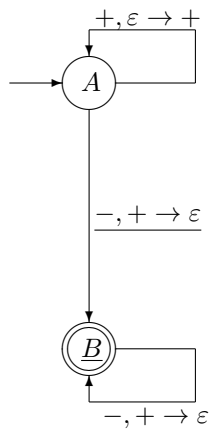
- we see the first $+$ and we push it into the stack; the automaton is still in the state A , with $+$ in the stack;



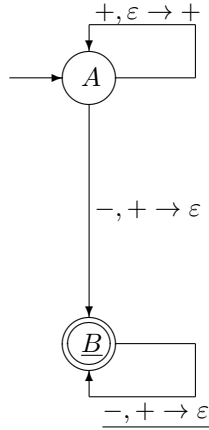
- then, we see the second $+$ and we push it into the stack; the automaton is still in the state A , with $++$ in the stack;



- now, we see a minus, so we pop the top $+$ from the stack and go to state B ; the stack now contains only one $+$:



- then, we see the last minus, so we pop $+$ from the stack; the automaton is still in the state B , and the stack is now empty:



Already reading all the symbols of the word $++--$, we are in the final state with an empty stack. Thus, this word is accepted.

To illustrate these transitions, let us list all the symbols we read, all the states that this automaton goes through, and under each state, the contents of the corresponding stack, with \rightarrow indicating transition corresponding to reading a symbol:

read		+		+		-		-	
state	A	\rightarrow	A	\rightarrow	A	\rightarrow	B	\rightarrow	B
stack		\rightarrow	+	\rightarrow	+	\rightarrow	+	\rightarrow	