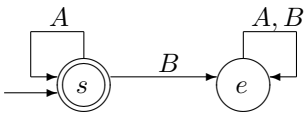
Solution to Homework 8

Background: In Homework 3, we considered the following automaton. This automaton has two states: s (= straight-A student) and e (= everyone else); s is the starting state, it is also the final state. The only two symbols are A and B.

- From s, A leads to s, and B to e.
- From e, any symbol leads back to e.

This automaton has the following form:



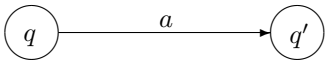
Tasks:

1. On the example of the automaton from Homework 3, show how the general algorithm will produce a context-free grammar that generates all the words accepted by this automaton – and only words generated by this automaton.
2. On the example of a word AAA accepted by this automaton, show how the tracing of acceptance of this word by the finite automaton can be translated into a generation of this same word by your context-free grammar.

Comment. In CFG, terminal symbols are small letters, so we will use a instead of A.

Solution to Task 1. The general algorithm for transforming FA into CFG is as follows:

- To each state q of the FA, introduce a new variable Q.
- The variable corresponding to the starting state will be the starting variable of the new CFG.
- For each transition of the finite automaton



we add a rule $Q \to aQ'$.

- For each final state f of the FA, we add a rule $F \to \varepsilon$.

By applying this general algorithm to this FA, we get a CFG with the starting variable S and the following rules:

$$S \to aS$$

$$S \to bE$$

$$E \to aE$$

$$E \to bE$$

$$S \to \varepsilon$$

Solution to Task 2. Derivations in this grammar follow, step-by-step, the way the original finite automaton accepts a word. The word AAA is accepted by the original finite automaton as follows:

- we start in the start state s; this corresponds to the starting variable S;
- then, we use the fact that once we are in the state s and we see the symbol A, then we move to the state s; this transition corresponds to the rule $S \to aS$, so the generation so far is:

$$\underline{S} \to aS;$$

- then, we use the fact that once we are in the state s and we see the symbol A, then we go to the state s; this transition corresponds to the rule $S \to aS$, so generation so far is

$$\underline{S} \to a\underline{S} \to aaS;$$

- then, we use the fact that once we are in the state s and we see the symbol A, then we go to the state s; this transition corresponds to the rule $S \to aS$, so generation so far is

$$\underline{S} \to a\underline{S} \to aa\underline{S} \to aaaS;$$

- we have read all the symbols of the word, and we are in the final state s; for the FA, this means that the word AAA is accepted; for CFG, we need to use the rule $S \to \varepsilon$ corresponding to the final state s; thus, we get the following derivation of the word AAA:

$$\underline{S} \to a\underline{S} \to aa\underline{S} \to aaa\underline{S} \to aaa.$$

So, we have indeed derived the word AAA in the grammar.