1-2. Prove that the language

\[ L = \{ s^{2n}q^nq^{n+2}r^2n \} = \{ qq, ssqqrr, sssssqqqqrr, \ldots \} \]

is not context-free.

3. Trace the following Turing machine on the example of the word 01:

- start, – → work, R (here, – means blank)
- work, 0 → 1, R
- work, 1 → 0, R
- work, – → back, L
- back, 0 → L
- back, 1 → L
- back, – → halt

Explain how each step will be represented if we interpret the Turing machine as a finite automaton with two stacks.

4. Design a Turing machine that adds 1000 (binary version of 8) to a binary number. Trace your Turing machine, step-by-step, on the example of the string 1011. Why in Turing machines (and in most actual computers) the representation of a binary number starts with the least significant digit?

5. The following finite automaton describes strings with at least one 1. This automaton has:

- the starting state s; this state means that we have not read any ones yet;
- the state s\(s\) meaning that we have read one 1; this state is also final; and
- the state s2 meaning that we have read two or more 1s; this state is also final.

Transitions are as follows:
from the state s, symbol 1 leads to state s1 and symbol 0 leads back to state s;

• from state s1, symbol 0 leads back to state s1 and 1 leads to state s2;

• from state s2, both symbols 0 and 1 lead back to the state s2.

Use the general algorithm to transform this finite automaton into a Turing machine. Show, step-by-step, how your Turing machine will accept the string 011.

6. Give the formal definition of a feasible algorithm. Give two examples different from what we had in class:

• an example of a computation time which is formally feasible, but not practically feasible, and

• an example of a computation time which is practically feasible but not formally feasible.

7. What is P? What is NP? What does it means for a problem to be NP-hard? NP-complete? Give brief definitions. Give an example of an NP-complete problem: explain what is the input, what is the desired output. Is P equal to NP?

8. Prove that the cubic root of 7 is not a rational number.

9. Formulate the halting problem. Prove that it is not possible to check whether a given program halts on given data.

10. Formulate Church-Turing thesis. Is it a mathematical theorem? Is it a statement about the physical world?