**Solution to Problem 14**

**Task.** For the \( LL(1) \) grammar that we studied in class, with rules \( S \to F \), \( S \to (S + F) \), and \( F \to a \), show how the word \(((a + a) + a) + a\) can be represented as \( uvxyz \) in accordance with the pumping lemma for context-free grammars. Show that the corresponding word \( uv^2xyz \) will be generated by this grammar.

**Solution.** The derivation of this string takes the following form:

\[
\begin{array}{c}
S \\
(S + F) \\
(S + F) \\
(S + F) \\
F \\
a
\end{array}
\]

In this derivation, we have several occurrences of the variable \( F \), but they are not on the same branch. The lowest pair of occurrences of the same variable is the lowest pair of occurrences of the variable \( S \):
Thus, the desired decomposition of this word into \( u, v, x, y, \) and \( z \) has the following form:

\[
\begin{align*}
S &\rightarrow (S + F) \\
&\rightarrow (S + F) \ a \\
F &\rightarrow a
\end{align*}
\]

So, here \( u = ((, v = (, x = a, y = +a), \) and \( z = +a + a). \) If we copy of the part between the two lowest occurrences of \( S \) to the lower occurrence, we conclude that the word \( uvvxyyz \) can be derived as follows: