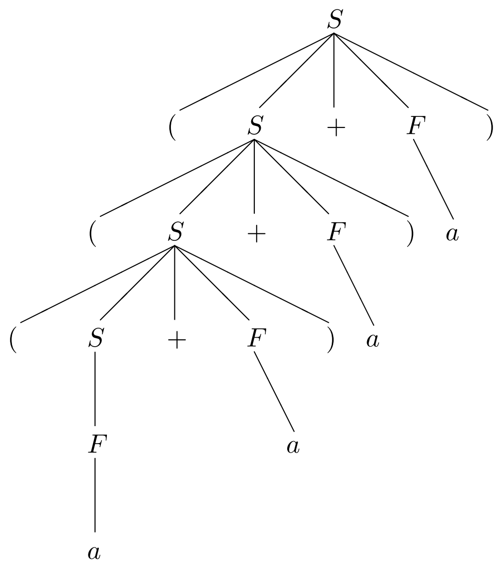
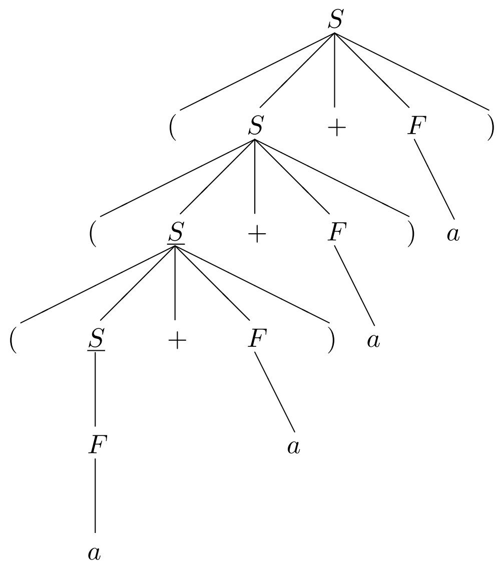
Solution to Problem 14

Task. For the $LL(1)$ grammar that we studied in class, with rules $S \to F$, $S \to (S + F)$, and $F \to a$, show how the word $(((a + a) + a) + a)$ can be represented as $uvxyz$ in accordance with the pumping lemma for context-free grammars. Show that the corresponding word $uvvxyyz$ will be generated by this grammar.

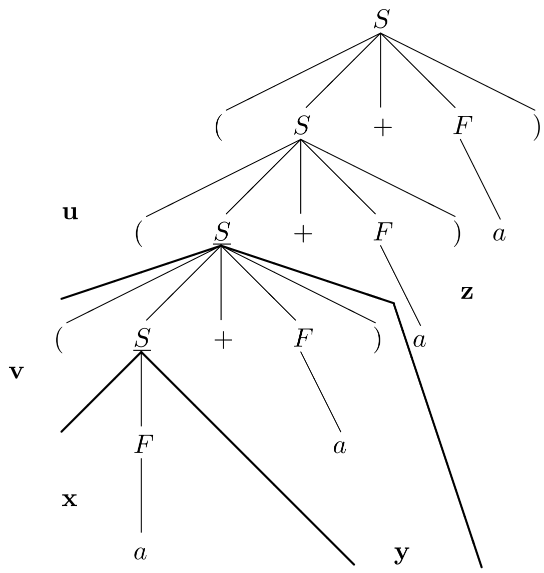
Solution. The derivation of this string takes the following form:



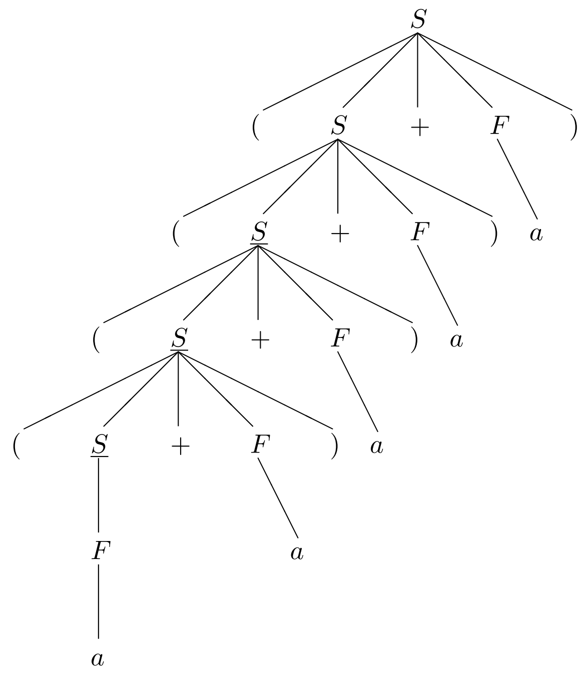
In this derivation, we have several occurrences of the variable F, but they are not on the same branch. The lowest pair of occurrences of the same variable is the lowest pair of occurrences of the variable S:



Thus, the desired decomposition of this word into u, v, x, y, and z has the following form:



So, here $u = (($, $v = ($, $x = a$, $y = +a)$, and $z = +a) + a)$. If we copy of the part between the two lowest occurrences of S to the lower occurrence, we conclude that the word $uvvxyyz$ can be derived as follows:


S
(S + F)
(S + F) a
(S + F) a
(S + F) a
F a
a