

## Solution to Homework Problem 16

**Homework Problem 16.** Design a Turing machine that, given a positive unary number  $n$ , add 3 to this number. Test it, step-by-step, on the example of  $n = 1$ .

**Idea.** This is similar to adding 1 to a unary number. The only difference is that when we reach the first blank space and replace it with 1, we are not done yet: we have one more 1 to add, etc.

**Solution.** Here are the rules for the Turing machine:

start,  $- \rightarrow R$ , moving  
 moving,  $1 \rightarrow R$   
 moving,  $- \rightarrow 1$ , R, adding2nd1  
 adding2nd1,  $- \rightarrow 1$ , R, adding3rd1  
 adding3rd1,  $- \rightarrow 1$ , L, back  
 back,  $1 \rightarrow L$   
 back,  $- \rightarrow \text{halt}$

**Tracing.**

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=	1	-	-	-	-	...		
<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">-</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;"><u>1</u></td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">-</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">-</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">-</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">-</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">...</td> </tr> </table>	-	<u>1</u>	-	-	-	-	...	moving
-	<u>1</u>	-	-	-	-	...		
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-	1	=	-	-	-	...		
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-	1	1	=	-	-	...		
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-	1	1	1	=	-	...		
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-	1	1	<u>1</u>	1	-	...		
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-	<u>1</u>	1	1	1	-	...		
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=	1	1	1	1	-	...		
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=	1	1	1	1	-	...		