

Solution to Homework Problem 17

Homework Problem 17. Design a Turing machine that, given a positive binary number n greater than or equal to 16, subtracts 16 from this number. Test it, step-by-step, on the example of $n = 18$.

Solution. Here are the rules for the Turing machine:

start, $- \rightarrow R$, skip1st
 skip1st, $0 \rightarrow R$, skip2nd
 skip1st, $1 \rightarrow R$, skip2nd
 skip2nd, $0 \rightarrow R$, skip3rd
 skip2nd, $1 \rightarrow R$, skip3rd
 skip3rd, $0 \rightarrow R$, skip4th
 skip3rd, $1 \rightarrow R$, skip4th
 skip4th, $0 \rightarrow R$, moving
 skip4th, $1 \rightarrow R$, moving
 moving, $0 \rightarrow 1, R$
 moving, $1 \rightarrow 0, L$, back
 back, $1 \rightarrow L$
 back, $0 \rightarrow L$
 back, $- \rightarrow \text{halt}$

Tracing. We start with the number $18_{10} = 10010_2$ which is represented as 01001.

-	0	1	0	0	1	-	...	start
-	<u>0</u>	1	0	0	1	-	...	skip1st
-	0	<u>1</u>	0	0	1	-	...	skip2nd
-	0	1	<u>0</u>	0	1	-	...	skip3rd
-	0	1	0	<u>0</u>	1	-	...	skip4th
-	0	1	0	0	<u>1</u>	-	...	moving
-	0	1	0	<u>0</u>	0	-	...	back
-	0	1	<u>0</u>	0	0	-	...	back
-	0	<u>1</u>	0	0	0	-	...	back
-	<u>0</u>	1	0	0	0	-	...	back

=	0	1	0	0	0	-	...	back
=	0	1	0	0	0	-	...	halt