

Solution to Homework Problem 19

Homework Problem 19. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

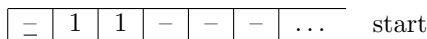
- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes $n + 1$ for a binary number $n = 3$, show, step-by-step:

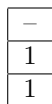
- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution.

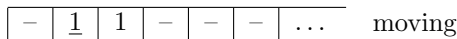
1. At first, we have the following configuration:



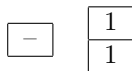
Here, the left stack is empty, and the right stack has the form



2. Then, the configuration changes to:



Here, the two stacks have the following form:



To get to this configuration, we pop the symbol - (meaning black space) from the right stack and push it into the left stack.

3. Then, the configuration changes to:

-	0	<u>1</u>	-	-	-	...
---	---	----------	---	---	---	-----

 moving

Here, the left stack has the following form:

0
-

and the right stack has the following form:

1

To get to this configuration, we replace 1 with 0, pop 0 from the right stack and push it into the left stack.

4. Then, the configuration changes to:

-	0	0	-	-	-	...
---	---	---	---	---	---	-----

 moving

Here, the left stack has the following form:

0
0
-

and the right stack has the following form:

-

To get to this configuration, we replace 1 with 0, pop 0 from the right stack and push it into the left stack, and - since nothing was left in the right stack - add -- (blank) to the right stack.

5. Then, the configuration changes to:

-	0	<u>0</u>	1	-	-	...
---	---	----------	---	---	---	-----

 back

Here, the left stack has the following form:

0
-

and the right stack has the following form:

0
1

To get to this configuration, we replace blank with 1, pop 1 from the left stack and push it into the right stack.

6. Then, the configuration changes to:

-	<u>0</u>	0	1	-	-	...
---	----------	---	---	---	---	-----

 back

Here, the left stack has the following form:

-

and the right stack has the following form:

0
0
1

To get to this configuration, we pop 0 from the left stack and push it into the right stack.

7. Then, the configuration changes to:

-	0	0	1	-	-	...
---	---	---	---	---	---	-----

back

Here, the left stack is empty, and the right stack has the following form:

-
0
0
1

To get to this configuration, we pop - from the left stack and push it into the right stack.

8. Then, the configuration changes to:

-	0	0	1	-	-	...
---	---	---	---	---	---	-----

halt

Here, the contents of the tape did not change, and the location of the head did not change, so the stacks remain the same.