Solutions to Homework 2

Task 2.1. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language $(A \cup B)(a \cup b)^*$.

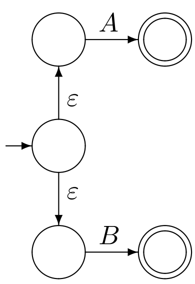
Reminder:

- A, B, a, and b are languages consisting of only one 1-symbol word each: A is a language consisting of a single 1-symbol word A; a is a language consisting of a single 1-symbol word a, etc.;
- for any two languages C and D, the notation CD means concatenation.

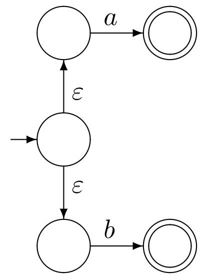
Solution. We start with the standard non-deterministic automata for recognizing:

- the language A – that consists of a single word A, and
- the language B – that consists of a single word B:

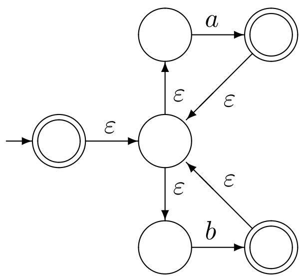
Then, we use the general algorithm for the union to design a non-deterministic automaton for recognizing the language $A \cup B$:



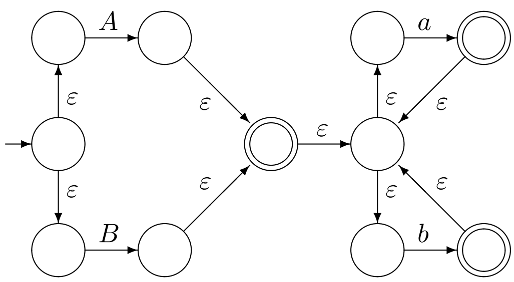
Similarly, we get a non-deterministic automaton for recognizing the language $a \cup b$:



Now, we apply a standard algorithm for the Kleene star, and we get the following non-deterministic automaton for $(a \cup b)^*$:

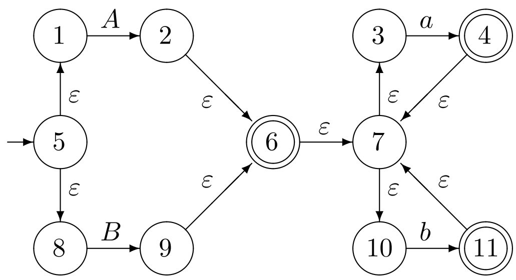


Now, we use the algorithm for concatenation for combine them: final states of the automaton for $A \cup B$ are no longer final, and from each of them, we add a jump to the starting state of the automaton for $(a \cup b)^*$:



Task 2.2. Transform the resulting non-deterministic finite automaton into a deterministic one.

Solution. Let us first enumerate the states of the resulting non-deterministic automaton.



In the beginning, before we see any symbol, we are in state 5, and we can also jump to states 1 and 8. Thus, before we see any symbols, we can be in one of the states 1, 5, and 8. This set $\{1, 5, 8\}$ is thus the starting state of the desired deterministic finite automaton. Checking where we can go from this state and from the resulting states when we see one of the symbols A, B, a, or b, we arrive at the following deterministic automaton.