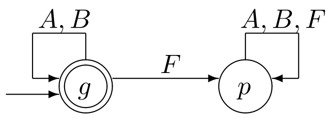
Solutions to Homework 3

Task: Apply the general algorithm for transforming the finite automaton into a regular language (i.e., a language described by a regular expression) to the following automaton.

This automaton has two states: g (good student) and p (student on probation); g is the starting state, it is also the final state. The only three symbols are A, B, and F.

- From g, A and B lead back to g, and F leads to p.
- From p, any symbol leads back to p.

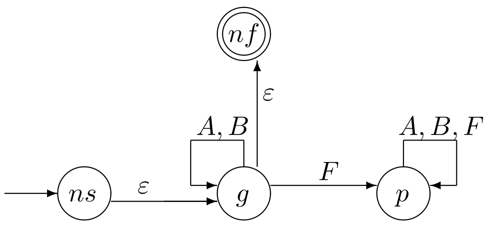
Solution. We start with the described automaton:



According to the general algorithm, first we add a new start state ns and a few final state nf, and we add jumps:

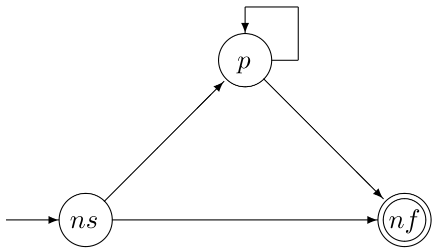
- from the old start state to the new start state, and
- from each old final state to the new final state.

As a result, we get the following automaton.



Then, we need to eliminate the two intermediate states g and p one by one. We can start with eliminating g or with eliminating p. Let us show what happens in both cases.

First version, when we first eliminate the state g. First, we draw all possible arrows:



Now, to find expressions to place at all these arrows, we will use the general formula

$$R'_{i,j} = R_{i,j} \cup (R_{i,k} R^*_{k,k} R_{k,j}),$$

where k is the state that we are eliminating, i.e., in this case, the state $k = s$.

By applying this formula, and by using simplification formulas described in the lecture, we get the following results:

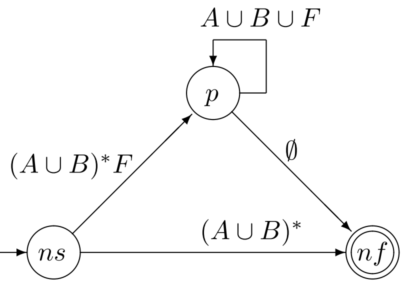
$$R'_{ns,p} = R_{ns,p} \cup (R_{ns,g} R^*_{g,g} R_{g,p}) = \emptyset \cup (\Lambda (A \cup B)^* F) = \emptyset \cup (A \cup B)^* F = (A \cup B)^* F;$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,g} R^*_{g,g} R_{g,nf}) = \emptyset \cup (\Lambda (A \cup B)^* \Lambda) = \emptyset \cup (A \cup B)^* = (A \cup B)^*;$$

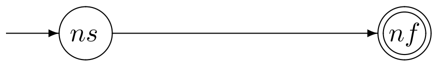
$$R'_{p,p} = R_{p,p} \cup (R_{p,g} R^*_{g,g} R_{g,p}) = (A \cup B \cup F) \cup (\emptyset \ldots) = (A \cup B \cup F) \cup \emptyset = A \cup B \cup F;$$

$$R'_{p,nf} = R_{p,nf} \cup (R_{p,g} R^*_{g,g} R_{g,nf}) = \emptyset \cup (\emptyset \ldots) = \emptyset \cup \emptyset = \emptyset.$$

Thus, the 3-state a-automaton takes the following form:



Now, all that remains to do is to go from here to the 2-state a-automaton by eliminating the remaining state p:



The final expression is the corresponding expression for $R'_{ns,nf}$:

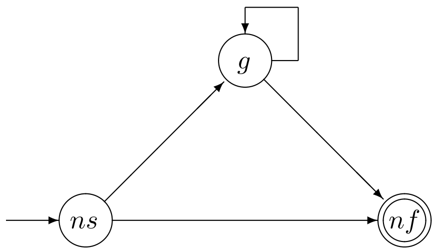
$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,p} R^*_{p,p} R_{p,nf}) =$$

$$(A \cup B)^* \cup ((A \cup B)^* F (A \cup B \cup F)^* \emptyset) = (A \cup B)^* \cup \emptyset = (A \cup B)^*.$$

The formula on the previous line is a regular expression corresponding to the original automaton.

First version – answer: $(A \cup B)^*$.

Second version, when we first eliminate the state p. First, we draw all possible arrows:



Now, to find expressions to place at all these arrows, we will use the general formula

$$R'_{i,j} = R_{i,j} \cup (R_{i,k} R^*_{k,k} R_{k,j}),$$

where k is the state that we are eliminating, i.e., in this case, the state $k = p$.

By applying this formula, and by using simplification formulas described in the lecture, we get the following results:

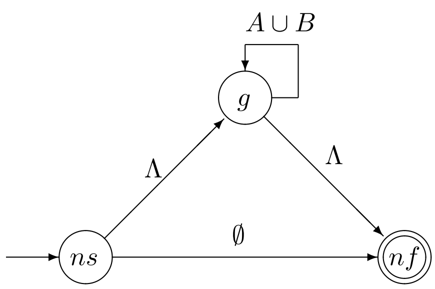
$$R'_{ns,g} = R_{ns,g} \cup (R_{ns,p} R^*_{p,p} R_{p,g}) = \Lambda \cup (\emptyset \ldots) = \Lambda \cup \emptyset = \Lambda;$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,p} R^*_{p,p} R_{p,nf}) = \emptyset \cup (\emptyset \ldots) = \emptyset \cup \emptyset = \emptyset;$$

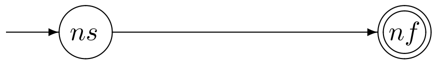
$$R'_{g,g} = R_{g,g} \cup (R_{g,p} R^*_{p,p} R_{p,g}) = (A \cup B) \cup (F (A \cup B \cup F)^* \emptyset) = A \cup B \cup \emptyset = A \cup B;$$

$$R'_{g,nf} = R_{g,nf} \cup (R_{g,p} R^*_{p,p} R_{p,nf}) = \Lambda \cup (F (A \cup B \cup F)^* \emptyset) = \Lambda \cup \emptyset = \Lambda.$$

Thus, the 3-state a-automaton takes the following form:



Now, all that remains to do is to go from here to the 2-state a-automaton by eliminating the remaining state g:



The final expression is the corresponding expression for $R'_{ns,nf}$:

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,g} R^*_{g,g} R_{g,nf}) =$$

$$\emptyset \cup (\Lambda(A \cup B)^* \Lambda) = (A \cup B)^*.$$

The formula $(A \cup B)^*$ is also a regular expression corresponding to the original automaton.

Second version – answer: $(A \cup B)^*$.