

## Solution to Problem 6

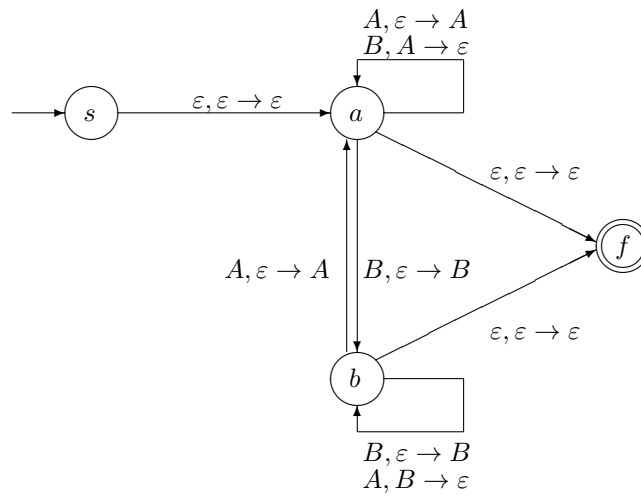
**Task:** Show, step by step, how the following pushdown automaton will recognize a sequence  $ABAB$ . This pushdown automaton has four states:

- the starting state  $s$ ,
- the state  $a$  meaning that the number of  $A$ s is larger than or equal to the number of  $B$ s,
- the state  $b$  meaning that the number of  $B$ s is larger than or equal to the number of  $A$ s, and
- the final state  $f$ .

The transitions are as follows:

- From  $s$  to  $a$ , the transition is  $\varepsilon, \varepsilon \rightarrow \varepsilon$ .
- From  $a$  to  $a$ , the transitions are:  $A, \varepsilon \rightarrow A$  and  $A, B \rightarrow \varepsilon$ .
- From  $a$  to  $b$ , the transition is  $B, \varepsilon \rightarrow B$ .
- From  $b$  to  $b$ , the transitions are:  $B, \varepsilon \rightarrow B$  and  $B, A \rightarrow \varepsilon$ .
- From  $b$  to  $a$ , the transition is  $A, \varepsilon \rightarrow A$ .
- From  $a$  to  $f$ , the transition is  $\varepsilon, \varepsilon \rightarrow \varepsilon$ .
- From  $b$  to  $f$ , the transition is  $\varepsilon, \varepsilon \rightarrow \varepsilon$ .

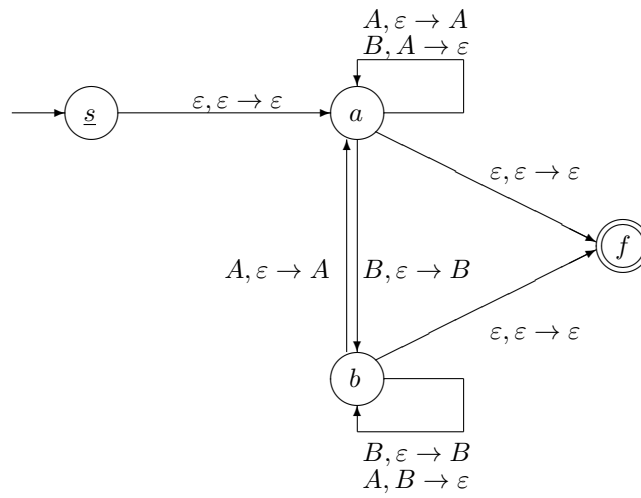
**Solution.** Our pushdown automaton has the following form:



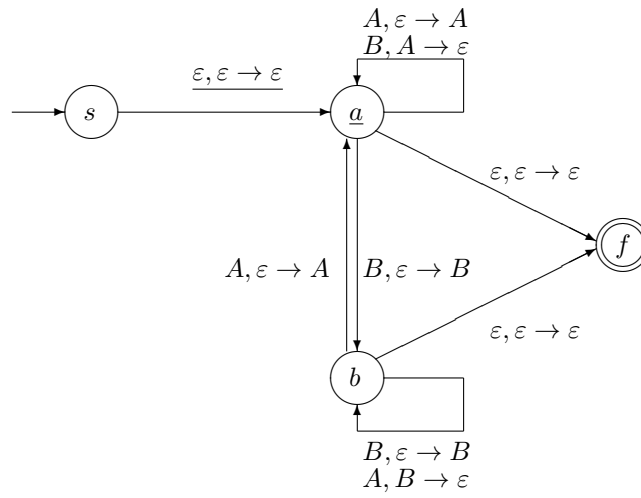
The main idea is that:

- first, we are in the state  $s$ ; we then jump to  $a$ ;
- then we see  $A$ , and we push  $A$  into the stack;
- then we see  $B$ , and we pop  $A$  from the stack;
- then we again see  $A$ , and we push  $A$  into the stack;
- then we again see  $B$ , and we pop  $A$  from the stack
- finally, we jump to the final state  $f$ .

So, we start in the starting state  $s$  with an empty stack.

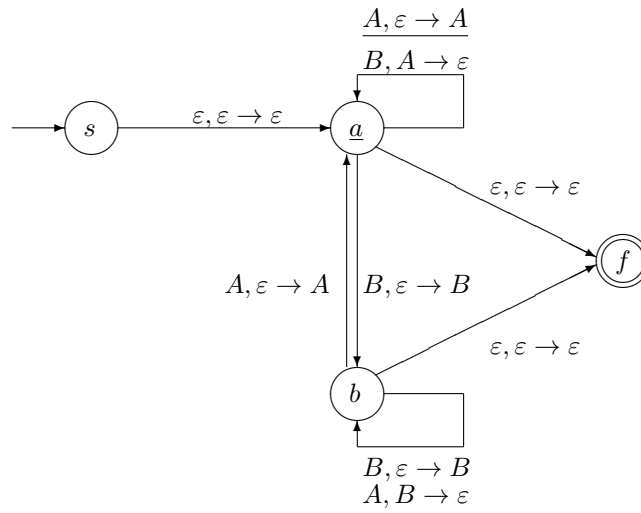


Then, we jump to the state  $a$ :



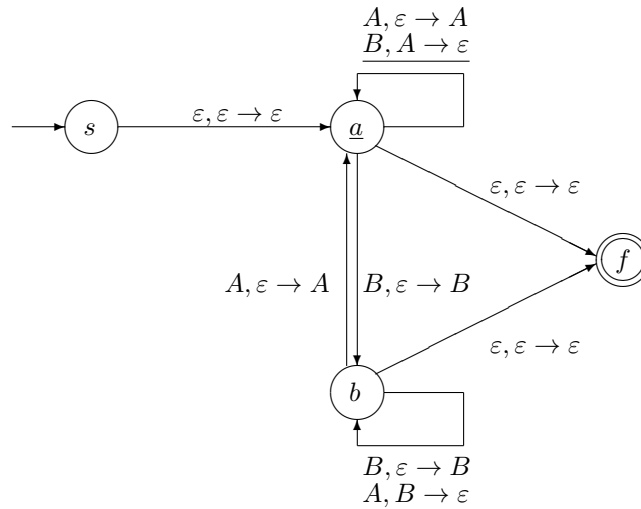
The stack is still empty.

Then, we see letter  $A$  from the word  $ABAB$ , and we push  $A$  into the stack:



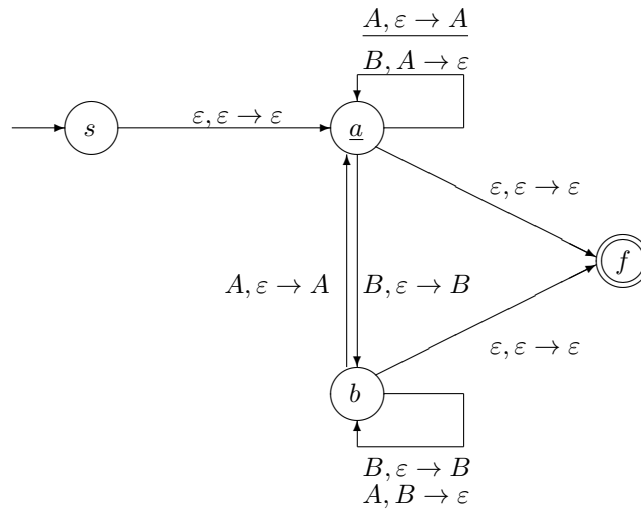
The stack has now only the letter  $A$ .

Then, we see the letter  $B$  from the word  $ABAB$ , and we pop  $A$  from the stack:



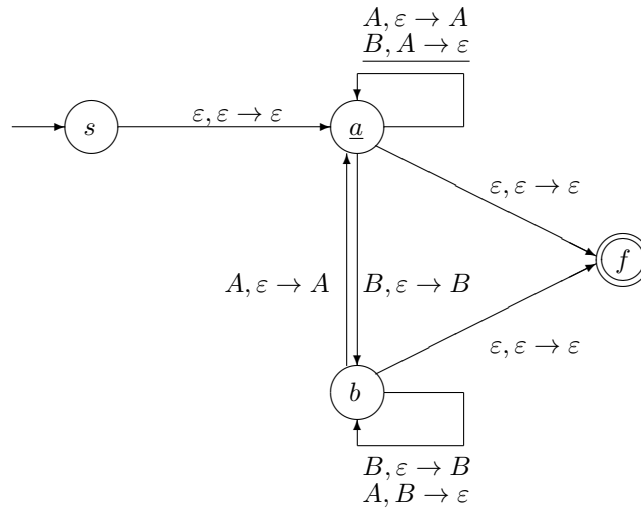
Now, the stack is empty.

Then, we see another letter  $A$  from the word  $ABAB$ , and we push  $A$  into the stack:



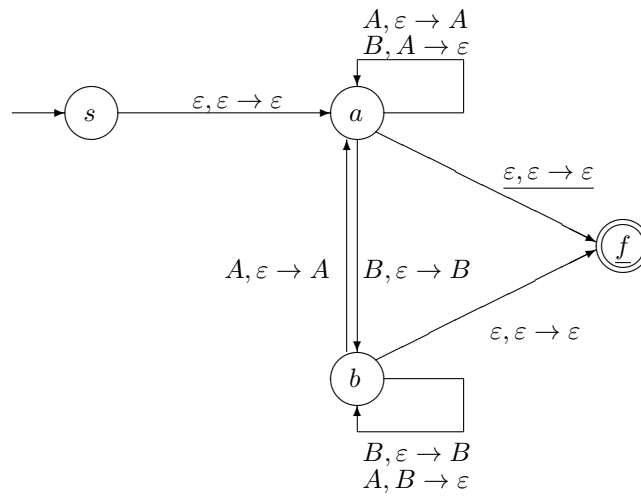
The stack has now only the letter  $A$ .

Then, we see the last letter  $B$  of the word  $ABAB$ , and we pop  $A$  from the stack:



Now, the stack is empty.

Finally, we jump into the final states  $f$ :



We have read all the symbols of the word  $ABAB$ , and we are in the final state with an empty stack. Thus, the word  $ABAB$  is accepted.

To illustrate these transitions, let us list all the symbols we read, all the states that this automaton goes through, and under each state, the contents of the corresponding stack, with  $\rightarrow$  indicating transition corresponding to reading a symbol:

read			$A$		$B$		$A$		$B$		
state	$s$	$a$	$\rightarrow$	$a$	$\rightarrow$	$a$	$\rightarrow$	$a$	$\rightarrow$	$a$	$f$
stack			$\rightarrow$	$A$	$\rightarrow$		$\rightarrow$	$A$	$\rightarrow$		