Solution to Problem 6

**Task:** Show, step by step, how the following pushdown automaton will recognize a sequence $ABAB$. This pushdown automaton has four states:

- the starting state $s$,
- the state $a$ meaning that the number of $A$s is larger than or equal to the number of $B$s,
- the state $b$ meaning that the number of $B$s is larger than or equal to the number of $A$s, and
- the final state $f$.

The transitions are as follows:

- From $s$ to $a$, the transition is $\varepsilon, \varepsilon \rightarrow \varepsilon$.
- From $a$ to $a$, the transitions are: $A, \varepsilon \rightarrow A$ and $A, B \rightarrow \varepsilon$.
- From $a$ to $b$, the transition is $B, \varepsilon \rightarrow B$.
- From $b$ to $b$, the transitions are: $B, \varepsilon \rightarrow B$ and $B, A \rightarrow \varepsilon$.
- From $b$ to $a$, the transition is $A, \varepsilon \rightarrow A$.
- From $a$ to $f$, the transition is $\varepsilon, \varepsilon \rightarrow \varepsilon$.
- From $b$ to $f$, the transition is $\varepsilon, \varepsilon \rightarrow \varepsilon$.

**Solution.** Our pushdown automaton has the following form:
The main idea is that:

- first, we are in the state $s$; we then jump to $a$;
- then we see $A$, and we push $A$ into the stack;
- then we see $B$, and we pop $A$ from the stack;
- then we again see $A$, and we push $A$ into the stack;
- then we again see $B$, and we pop $A$ from the stack
- finally, we jump to the final state $f$.

So, we start in the starting state $s$ with an empty stack.
Then, we jump to the state $a$:

The stack is still empty.

Then, we see letter $A$ from the word $A B A B$, and we push $A$ into the stack:
The stack has now only the letter $A$.
Then, we see the letter $B$ from the word $ABAB$, and we pop $A$ from the stack:

Now, the stack is empty.
Then, we see another letter $A$ from the word $ABAB$, and we push $A$ into the stack:
The stack has now only the letter $A$.
Then, we see the last letter $B$ of the word $ABAB$, and we pop $A$ from the stack:

Now, the stack is empty.
Finally, we jump into the final stats $f$: 
We have read all the symbols of the word $ABAB$, and we are in the final state with an empty stack. Thus, the word $ABAB$ is accepted.

To illustrate these transitions, let us list all the symbols we read, all the states that this automaton goes through, and under each state, the contents of the corresponding stack, with $\rightarrow$ indicating transition corresponding to reading a symbol:

<table>
<thead>
<tr>
<th>read</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>state</td>
<td>s</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>stack</td>
<td>$\rightarrow$</td>
<td>$A$</td>
<td>$\rightarrow$</td>
<td>$A$</td>
</tr>
</tbody>
</table>