

Solution to Homework 7

Task: Show, step by step, how the grammar with rules $S \rightarrow \varepsilon$, $S \rightarrow ABS$, $S \rightarrow ASB$, $S \rightarrow SAB$, $S \rightarrow SBA$, $S \rightarrow BSA$, $S \rightarrow SBA$, and $S \rightarrow SS$ will generate the word $ABAB$.

Solution. In this language, there is only one variable S , so this variable is a starting variable.

Option 1. Since the word $ABAB$ starts with A and ends with B , one possibility is to use the rule $S \rightarrow ASB$, meaning that if we have a word from this language and we add A in front and B at the end, we still get the word from the language. Our word $ABAB$ is obtained this way from the word AB , so to derive the word $ABAB$, it is sufficient to show that the auxiliary word BA belongs to the language.

$$\underline{S} \rightarrow A\underline{S}B \rightarrow \dots \rightarrow ABAB.$$

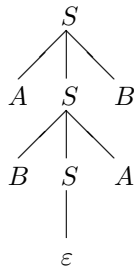
The word BA starts with B and ends with A , so one possibility is to use the rule $S \rightarrow BSA$, meaning that if we have a word from this language and we add B in front and A at the end, we still get the word from the language. The word BA is obtained this way from the empty string Λ , so to derive the word BA , it is sufficient to show that the auxiliary word Λ belongs to the language.

$$\underline{S} \rightarrow A\underline{S}B \rightarrow AB\underline{S}BB \rightarrow \dots \rightarrow ABAB.$$

The empty string does belong to the language: this is what the rule $S \rightarrow \varepsilon$ says. Thus, we have the desired derivation:

$$\underline{S} \rightarrow A\underline{S}B \rightarrow AB\underline{S}BB \rightarrow ABAB.$$

So, we got the following derivation tree:



Option 2. We can notice that $ABAB$ consists of the word AB repeated twice. The rule $S \rightarrow SS$ means that if we have two words from the language, then their concatenation belongs to the language. Thus, to prove that the word $ABAB$ belongs to the language, it is sufficient to prove that the word AB belongs to this language. This follows from the rules $S \rightarrow ASB$ and $S \rightarrow \varepsilon$. Thus, we have

$$\underline{S} \rightarrow \underline{SS} \rightarrow \underline{ASBS} \rightarrow \underline{ABS} \rightarrow \underline{ABASB} \rightarrow \underline{ABAB}.$$

So, we got the following derivation tree:

