Solution to Homework 8

Background: In Homework 3, we considered the following automaton. This automaton has two states: g (good student) and p (student on probation); g is the starting state, it is also the final state. The only three symbols are A, B, and F.

- From g, A and B lead back to g, and F leads to p.
- From p, any symbol leads back to p.

This automaton has the following form:

Tasks:

1. On the example of the automaton from Homework 3, show how the general algorithm will produce a context-free grammar that generates all the words accepted by this automaton – and only words generated by this automaton.

2. On the example of a word ABB accepted by this automaton, show how the tracing of acceptance of this word by the finite automaton can be translated into a generation of this same word by your context-free grammar.

Comment. In CFG, terminal symbols are small letters, so we will use a instead of A.

Solution to Task 1. The general algorithm for transforming FA into CFG is as follows:

- To each state q of the FA, introduce a new variable Q.
- The variable corresponding to the starting state will be the starting variable of the new CFG.
• For each transition of the finite automaton

\[ q \xrightarrow{a} q' \]

we add a rule \( Q \rightarrow aQ' \).

• For each final state \( f \) of the FA, we add a rule \( F \rightarrow \varepsilon \).

By applying this general algorithm to this FA, we get a CFG with the starting variable \( G \) and the following rules:

\[
G \rightarrow aG \\
G \rightarrow aG \\
G \rightarrow fP \\
P \rightarrow aP \\
P \rightarrow bP \\
P \rightarrow fP \\
G \rightarrow \varepsilon
\]

**Solution to Task 2.** Derivations in this grammar follow, step-by-step, the way the original finite automaton accepts a word. The word \( ABB \) is accepted by the original finite automaton as follows:

• we start in the start state \( g \); this corresponds to the starting variable \( G \);

• then, we use the fact that once we are in the state \( g \) and we see the symbol \( A \), then we move to the state \( g \); this transition corresponds to the rule \( G \rightarrow aG \), so the generation so far is:

\[
G \rightarrow aG
\]

• then, we use the fact that once we are in the state \( g \) and we see the symbol \( B \), then we go to the state \( g \); this transition corresponds to the rule \( G \rightarrow bG \), so generation so far is

\[
G \rightarrow aG \rightarrow abG
\]

• then, we use the fact that once we are in the state \( g \) and we see the symbol \( B \), then we go to the state \( g \); this transition corresponds to the rule \( G \rightarrow bG \), so generation so far is

\[
G \rightarrow aG \rightarrow abG \rightarrow abG
\]
we have read all the symbols of the word, and we are in the final state $g$; for the FA, this means that the word $ABB$ is accepted; for CFG, we need to use the rule $G \rightarrow \varepsilon$ corresponding to the final state $g$; thus, we get the following derivation of the word $ABB$:

$$G \rightarrow aG \rightarrow abG \rightarrow abbg \rightarrow abb.$$ 

So, we have indeed derived the word $ABB$ in the grammar.