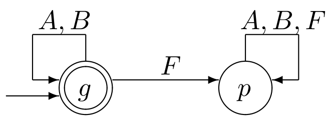
Solution to Homework 8

Background: In Homework 3, we considered the following automaton. This automaton has two states: g (good student) and p (student on probation); g is the starting state, it is also the final state. The only three symbols are A, B, and F.

- From g, A and B lead back to g, and F leads to p.
- From p, any symbol leads back to p.

This automaton has the following form:



Tasks:

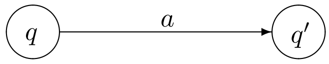
1. On the example of the automaton from Homework 3, show how the general algorithm will produce a context-free grammar that generates all the words accepted by this automaton – and only words generated by this automaton.
2. On the example of a word ABB accepted by this automaton, show how the tracing of acceptance of this word by the finite automaton can be translated into a generation of this same word by your context-free grammar.

Comment. In CFG, terminal symbols are small letters, so we will use a instead of A.

Solution to Task 1. The general algorithm for transforming FA into CFG is as follows:

- To each state q of the FA, introduce a new variable Q.
- The variable corresponding to the starting state will be the starting variable of the new CFG.

- For each transition of the finite automaton



  we add a rule $Q \to aQ'$.

- For each final state f of the FA, we add a rule $F \to \varepsilon$.

By applying this general algorithm to this FA, we get a CFG with the starting variable G and the following rules:

$$G \to aG$$

$$G \to aG$$

$$G \to fP$$

$$P \to aP$$

$$P \to bP$$

$$P \to fP$$

$$G \to \varepsilon$$

Solution to Task 2. Derivations in this grammar follow, step-by-step, the way the original finite automaton accepts a word. The word ABB is accepted by the original finite automaton as follows:

- we start in the start state g; this corresponds to the starting variable G;

- then, we use the fact that once we are in the state g and we see the symbol A, then we move to the state g; this transition corresponds to the rule $G \to aG$, so the generation so far is:

$$\underline{G} \to aG;$$

- then, we use the fact that once we are in the state g and we see the symbol B, then we go to the state g; this transition corresponds to the rule $G \to bG$, so generation so far is

$$\underline{G} \to a\underline{G} \to abG;$$

- then, we use the fact that once we are in the state g and we see the symbol B, then we go to the state g; this transition corresponds to the rule $G \to bG$, so generation so far is

$$\underline{G} \to a\underline{G} \to ab\underline{G} \to abbG;$$

- we have read all the symbols of the word, and we are in the final state g; for the FA, this means that the word ABB is accepted; for CFG, we need to use the rule $G \to \varepsilon$ corresponding to the final state g; thus, we get the following derivation of the word ABB:

$$\underline{G} \to a\underline{G} \to ab\underline{G} \to abb\underline{G} \to abb.$$

So, we have indeed derived the word ABB in the grammar.