Automata, Computability, and Formal Languages
Spring 2022, Test 3

1-2. Prove that the language

$$L = \{a^n b^n c^n d^n\} = \{\Lambda, abcd, aabbccdd, \ldots\}$$

is not context-free.

3. Trace the following Turing machine on the example of the word 01:

- start, – → swap, R (here, – means blank)
- swap, 0 → 1, R
- swap, 1 → 0, R
- swap, – → back, L
- back, 0 → 1, L
- back, 1 → 0, L
- back, – → halt

Explain how each step will be represented if we interpret the Turing machine as a finite automaton with two stacks.

4. Design a Turing machine that adds 100 (binary version of 4) to a binary number. Trace your Turing machine, step-by-step, on the example of the string 1101 (corresponding to the number 1011). Why in Turing machines (and in most actual computers) the representation of a binary number starts with the least significant digit?

5. The following finite automaton describes strings with odd number of 0s.

- the starting state e; this state means that we have read an even number of 0s; and
- the final state f meaning that we have read an off number of 0s.

Transitions are as follows:

- from the state e, symbol 0 leads to the state f and symbol 1 leads back to the state e;

- from the state f, symbol 0 leads to the state e and 1 leads back to the state f.

Use the general algorithm to transform this finite automaton into a Turing machine. Show, step-by-step, how your Turing machine will accept the string 101.

6. Give the formal definition of a feasible algorithm, and an explanation what practically feasible means. Give two examples different from what we had in class:

- an example of a computation time which is formally feasible, but not practically feasible, and
- an example of a computation time which is practically feasible but not formally feasible.

7. What is P? What is NP? What does it means for a problem to be NP-hard? NP-complete? Give brief definitions. Give an example of an NP-complete problem: explain what is the input, what is the desired output. Is P equal to NP?

8. Prove that the cubic root of 6 is not a rational number.

9. Formulate the halting problem. Prove that it is not possible to check whether a given program halts on given data.

10. Formulate Church-Turing thesis. Is it a mathematical theorem? Is it a statement about the physical world?