Solution to Problem 12

**Task.** For the grammar described in Homework 7, show how the word +110 can be represented as \( uvxyz \) in accordance with the pumping lemma for context-free grammars. Show that the corresponding word \( uv^2xyz \) will be generated by this grammar.

**Solution.** The derivation of this string takes the following form:

The lowest pair of occurrences of the same variable is the lowest pair of occurrences of the variable \( U \):
Thus, the desired decomposition of this word into $u$, $v$, $x$, $y$, and $z$ has the following form:

So, here $u = +1$, $v = 1$, $x = 0$, and $y = z = \varepsilon$. If we copy of the part between the two lowest occurrences of $U$ to the lower occurrence, we conclude that the word $uvvxyyz$ can be derived as follows: