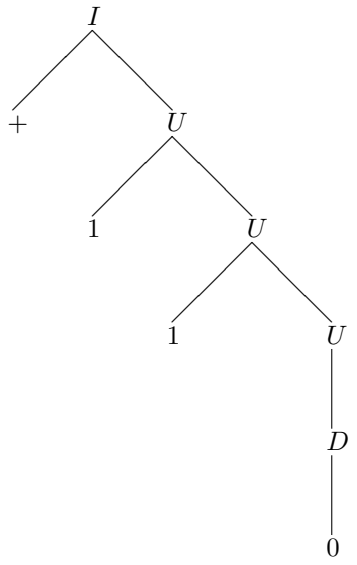
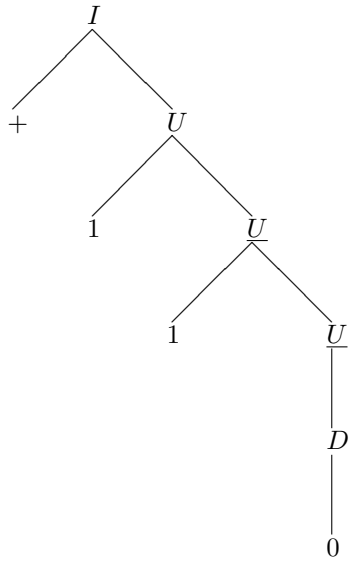
Solution to Problem 12

Task. For the grammar described in Homework 7, show how the word +110 can be represented as $uvxyz$ in accordance with the pumping lemma for context-free grammars. Show that the corresponding word $uvvxyyz$ will be generated by this grammar.

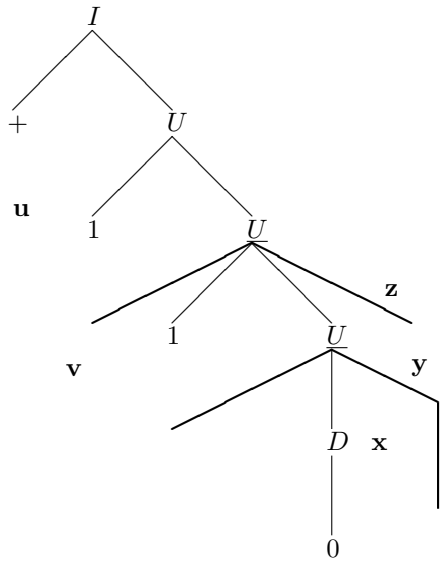
Solution. The derivation of this string takes the following form:



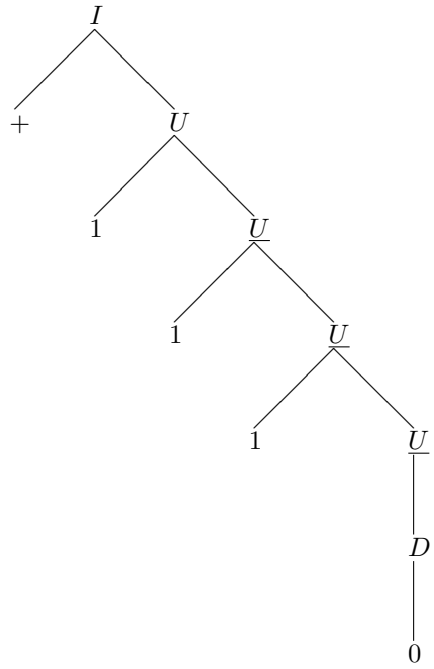
The lowest pair of occurrences of the same variable is the lowest pair of occurrences of the variable U:



Thus, the desired decomposition of this word into u, v, x, y, and z has the following form:



So, here $u = +1$, $v = 1$, $x = 0$, and $y = z = \varepsilon$. If we copy of the part between the two lowest occurrences of U to the lower occurrence, we conclude that the word $uvvxyyz$ can be derived as follows:


I
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U
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