Solution to Homework Problem 15

Task. Design a Turing machine that, given a positive unary number $n \geq 2$, subtracts 2 from this number. Test it, step-by-step, on the example of $n = 3$.

Idea. This is similar to subtracting 1 from a unary number. The only difference is that when we reach the first blank space – meaning that we have over all the 1s, we need to get back and delete two 1s. When we delete the first 1, we are not done yet: we have one more 1 to delete. Then, we go back.

Solution. Here are the rules for the Turing machine:
start, – → R, moving
moving, 1 → R
moving, – → deleting1st, L
deleting1st, 1 → –, deleting2nd, L
deleting2nd, 1 → –, back, L
back, 1 → L
back, – → halt

Tracing.

$\underline{-}$	1	1	1	–	–	...	start
–	$\underline{1}$	1	1	–	–	...	working
–	1	$\underline{1}$	1	–	–	...	working
–	1	1	$\underline{1}$	–	–	...	working
–	1	1	1	$\underline{-}$	–	...	working
–	1	1	$\underline{1}$	–	–	...	deleting1st
–	1	$\underline{1}$	–	–	–	...	deleting2nd
–	$\underline{1}$	–	–	–	–	...	back
$\underline{-}$	1	–	–	–	–	...	back
$\underline{-}$	1	–	–	–	–	...	halt