Solution to Homework Problem 16

**Task.** Design a Turing machine that, given a positive binary number $n$ greater than or equal to 16, adds 16 from this number. Test it, step-by-step, on the example of $n = 3$.

**Solution.** Here are the rules for the Turing machine:

- **start**, $\rightarrow R$, skip1st
- skip1st, 0 $\rightarrow$ R, skip2nd
- skip1st, 1 $\rightarrow$ R, skip2nd
- skip1st, $\rightarrow$ 0
- skip2nd, 0 $\rightarrow$ R, skip3rd
- skip2nd, 1 $\rightarrow$ R, skip3rd
- skip2nd, $\rightarrow$ 0
- skip3rd, 0 $\rightarrow$ R, skip4th
- skip3rd, 1 $\rightarrow$ R, skip4th
- skip3rd, $\rightarrow$ 0
- skip4th, 0 $\rightarrow$ R, moving
- skip4th, 1 $\rightarrow$ R, moving
- skip4th, $\rightarrow$ 0
- moving, 1 $\rightarrow$ 0, R
- moving, 0 $\rightarrow$ 1, L, back
- moving, $\rightarrow$ 1, L, back
- back, 1 $\rightarrow$ L
- back, 0 $\rightarrow$ L
- back, $\rightarrow$ halt

**Tracing.** We start with the number $3_{10} = 11_2$ which is represented as 01001.

```
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  start
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  skip1st
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  skip2nd
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  skip3rd
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  skip4th
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  moving
  - 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  back

1
```
| 1 1 0 0 1 | back |
| 1 1 0 0 1 | back |
| 1 1 0 0 1 | back |
| 1 1 0 0 1 | back |
| 1 1 0 0 1 | halt |