Solution to Homework 1

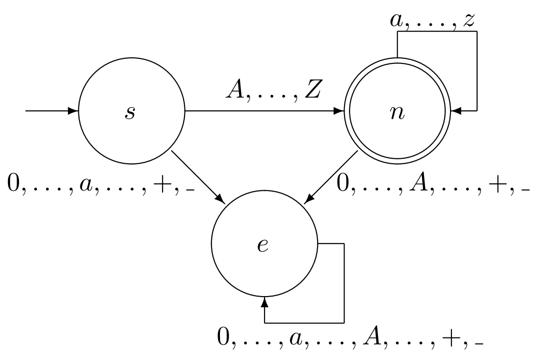
Task 1: general description. In class, we designed automata for recognizing integers and real numbers.

Task 1.1. Use the same ideas to describe an automaton for recognizing people's names. A general name should start with a capital (= uppercase) letter, all other letters should be small (= lowercase).

A natural idea is to have 3 states: start (s), correct name (n), and error (e). Start is the starting state, n is the only final state. The transitions are as follows:

- from s, any capital letter $A, \ldots, Z$ lead to n, every other symbol leads to e;
- from n, any small letter leads back to n, every other symbol leads to e;
- from e, every symbol leads back to e.

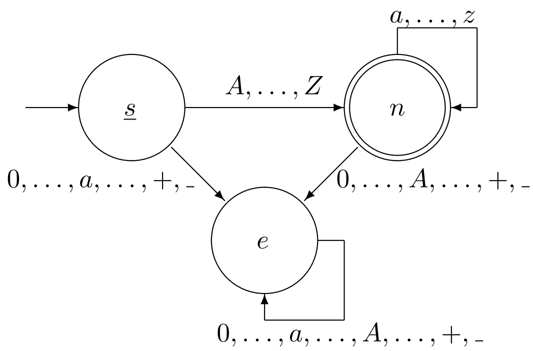
Solution. The desired automaton takes the following form:



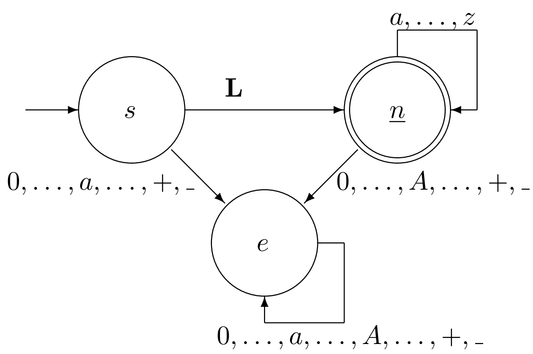
Task 1.2. Trace, step-by-step, how the finite automaton from Part 1.1 will check whether the following two words (sequences of symbols) are correct names for Java constants:

- the word Luc (which this automaton should accept) and
- the word LUC (which this automaton should reject).

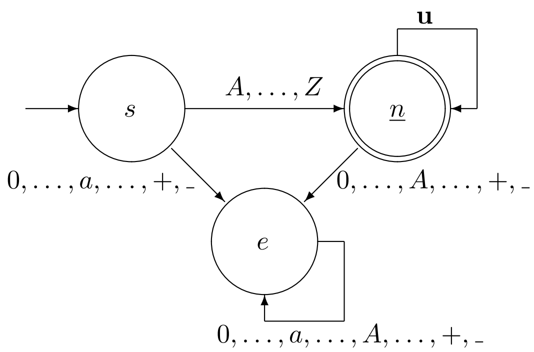
Solution. Let us trace how this automaton will accept the word Luc We are originally in the state s:



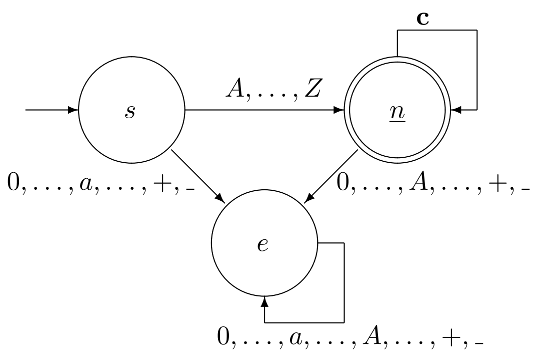
Then, we read the first letter L of the word **L**uc, so we move to state n:



Then, we read the second letter u of the word L**u**c, and we stay in the state n:

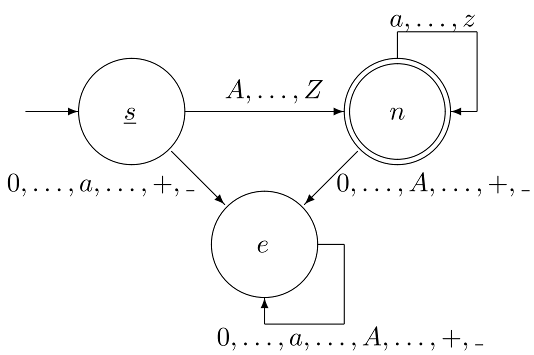


Then, we read the third symbol c of the word $Lu\mathbf{c}$, and we stay in the state n:

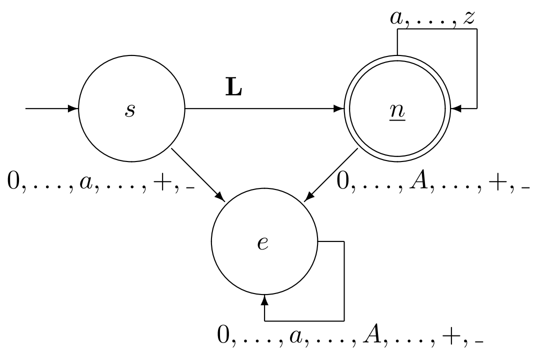


The word is read, we are in the final state, so the word Luc is accepted.

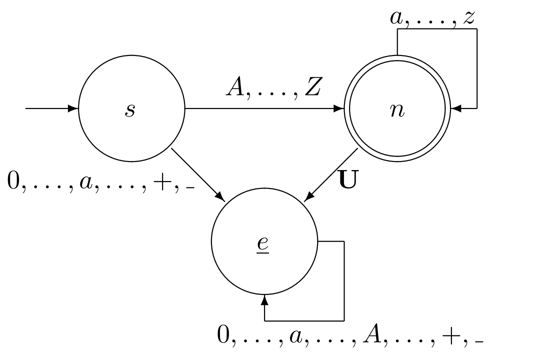
Let us now trace how the automaton will react to the word LUC. We also start in the start state s:



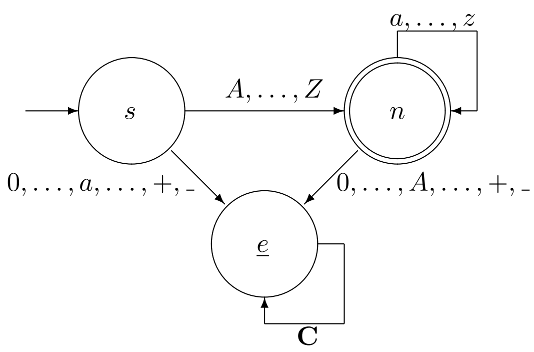
Then, we read the first letter L of the word $\mathbf{L}UC$, so we move to the state n;



After that, we read the second symbol U of the word $L\mathbf{U}C$ and move to state e:



Then, we read the last symbol **C** of the word $LUC\mathbf{C}$ and stay in the state e:



We have read all the symbols, we are in the state e which is not final, so the word LUC is not accepted.

Task 1.3. Write down the tuple $\langle Q, \Sigma, \delta, q_0, F\rangle$ corresponding to the automaton from Part 1.1:

- Q is the set of all the states,
- Σ is the alphabet, i.e., the set of all the symbols that this automaton can encounter; for simplicity, consider only four symbols: the plus sign, letters a and A, and an underscore;
- $\delta : Q \times \Sigma \to Q$ is the function that describes, for each state q and for each symbol s, the state $\delta(q, s)$ to which the automaton that was originally in the state q moves when it sees the symbol s (you do not need to describe all possible transitions this way, just describe two of them);
- q_0 is the staring state, and
- F is the set of all final states.

Solution. $Q = \{s, n, e\}$, $\Sigma = \{a, A, 1\}$, $q_0 = s$, $F = \{n\}$, and the transition function δ is described by the following table:

	a	A	1
s	e	n	e
n	n	e	e
e	e	e	e

Task 1.4. Apply the general algorithm for union and intersection to:

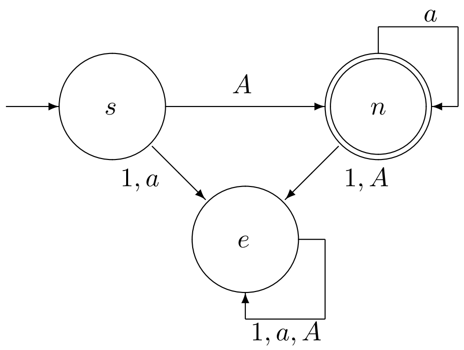
- the automaton from Part 1.1 as Automaton A and
- an automaton for recognizing Java names for classes as Automaton B.

In Java, a name for a class should start with a capital letter, all other symbols can be letters (small or capital), digits, or an underscore symbol. A natural idea is to also have 3 states: start (s), correct class name (c), and error (e). Start is the starting state, c is the only final state. The transitions are as follows:

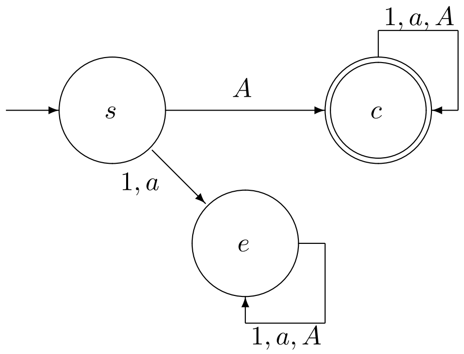
- from s, any capital letter $A, \ldots, Z$ lead to c, every other symbol leads to e;
- from c, any small letter a, $\ldots$, z, digit, or underscore leads back to c, every other symbol leads to e;
- from e, every symbol leads back to e.

For simplicity, in your automaton for recognizing the union and intersection of the two languages, feel free to assume that you only have symbols a, A, and 1.

Solution. If we limit ourselves to these 3 symbols, then the Automaton A takes the following form:



The Automaton B has the following form:



In the beginning, before we see any symbols, both automata are in the state s, so the combined automaton is in the state (s, s). Then:

- if we read A, Automaton A goes into state n and automaton B goes into state c, so we go into the state (n, c);
- if we read 1 or a, both automata go into the e states, so the combined automaton goes into the state (e, e).

We can similarly describe transitions from these three new states. As a result, we get the following automaton: