Solution to Homework Problem 20

Homework Problem 20. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes $n + 2$ for a binary number $n = 3$, show, step-by-step:

- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution. The rules of this Turing machine are as follows:
start, $-\to$ skip, R
skip, $0 \to$ moving, R
skip, $1 \to$ moving, R
moving, $1 \to 0$, R
moving, $0 \to 1$, L, back
moving, $-\to 1$, L, back
back, $0 \to$ L
back, $1 \to$ L
back, $-\to$ halt

1. At first, we have the following configuration:

$\underline{-}$	1	1	−	−	−	...	start

Here, the left stack is empty, and the right stack has the form

−
1
1

2. Then, the configuration changes to:

–	$\underline{1}$	1	–	–	–	. . .	skip

Here, the two stacks have the following form:

–	1
	1

To get to this configuration, we pop the symbol – (meaning black space) from the right stack and push it into the left stack.

3. Then, the configuration changes to:

–	1	$\underline{1}$	–	–	–	. . .	moving

Here, the left stack has the following form:

1	and the right stack has the following form:
–	

1

To get to this configuration, we pop 1 from the right stack and push it into the left stack.

4. Then, the configuration changes to:

–	1	0	$\underline{-}$	–	–	. . .	moving

Here, the left stack has the following form:

0	
1	and the right stack has the following form:
–	

–

To get to this configuration, we replace 1 with 0, pop 0 from the right stack and push it into the left stack, and – since nothing was left in the right stack – add –– (blank) to the right stack.

5. Then, the configuration changes to:

–	1	$\underline{0}$	1	–	–	. . .	back

Here, the left stack has the following form:

1	and the right stack has the following form:
–	

0
1

To get to this configuration, we replace blank with 1, pop 1 from the left stack and push it into the right stack.

6. Then, the configuration changes to:

–	$\underline{1}$	0	1	–	–	...	back

Here, the left stack has the following form:

–

and the right stack has the following form:

1
0
1

To get to this configuration, we pop 1 from the left stack and push it into the right stack.

7. Then, the configuration changes to:

$\underline{–}$	1	0	1	–	–	...	back

Here, the left stack is empty, and the right stack has the following form:

–
1
0
1

To get to this configuration, we pop – from the left stack and push it into the right stack.

8. Then, the configuration changes to:

$\underline{–}$	1	0	1	–	–	...	halt

Here, the contents of the tape did not change, and the location of the head did not change, so the stacks remain the same.