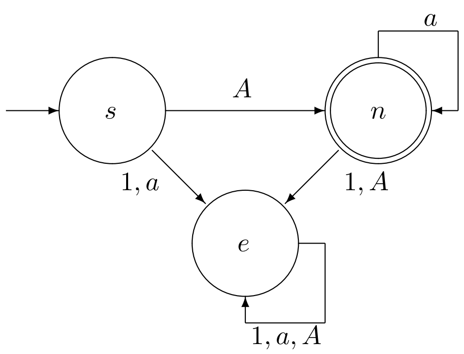
Solutions to Homework 3

Task: Apply the general algorithm for transforming the finite automaton into a regular language (i.e., a language described by a regular expression) to Automaton from Problem 1.1. For simplicity, assume that we only have symbols A, a, and 1. Eliminate first the error state, then the start state, and finally, the state n.

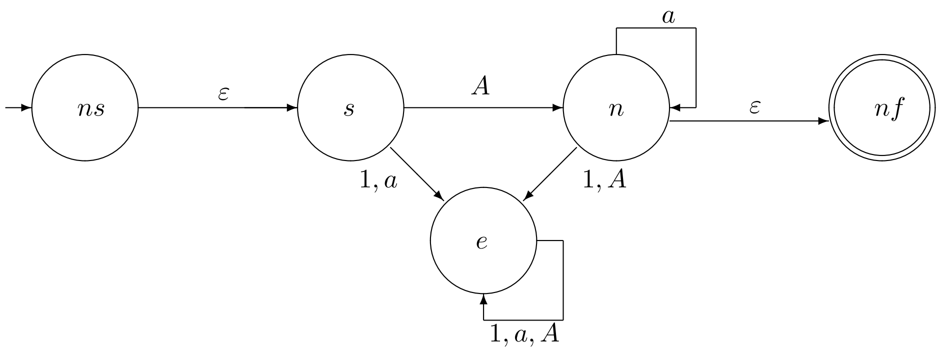
Solution. We start with the described automaton:



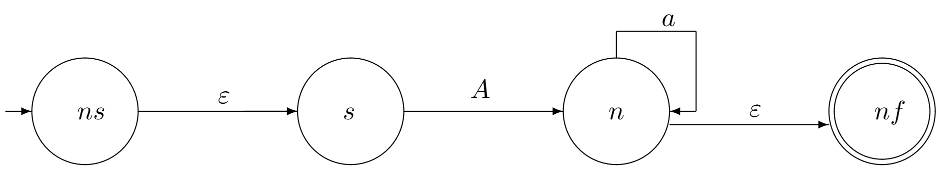
According to the general algorithm, first we add a new start state ns and a few final state nf, and we add jumps:

- from the old start state to the new start state, and
- from each old final state to the new final state.

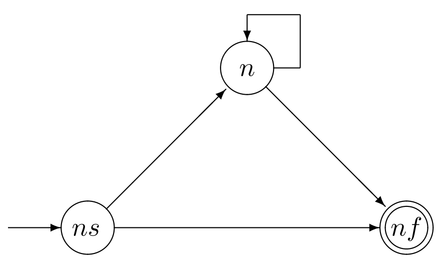
As a result, we get the following automaton:



Eliminating the sink state. Then, we need to eliminate the intermediate states one by one. Let us start with eliminating the sink state e. As a result, for all other states i and j, we get $R'_{i,j} = R_{i,j} \cup (R_{i,e}R^*_{e,e}R_{e,j})$. By definition of a sink state, it has no arrows going from it to any other states. Thus, we always have $R_{e,j} = \emptyset$. Concatenation with the empty set R^*e, j is empty set, so we always have $R_{i,e}R^*_{e,e}R_{e,j} = \emptyset$. Union of any set with the empty set is that same original set, so we have $R'_{i,j} = R_{i,j}$. Thus, we get the following simplified automaton.



Eliminating the start state. First, we draw all possible arrows:



Now, to find expressions to place at all these arrows, we will use the general formula

$$R'_{i,j} = R_{i,j} \cup (R_{i,k}R^*_{k,k}R_{k,j}),$$

where k is the state that we are eliminating, i.e., in this case, the state $k = s$.

By applying this formula, and by using simplification formulas described in the lecture, we get the following results:

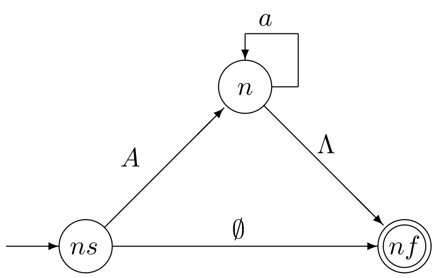
$$R'_{ns,n} = R_{ns,n} \cup (R_{ns,s}R^*_{s,s}R_{s,n}) = \emptyset \cup (\Lambda\emptyset^* A) = \emptyset \cup (\Lambda\Lambda A) = \emptyset \cup A = A;$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,s}R^*_{s,s}R_{s,nf}) = \emptyset \cup (\Lambda\emptyset^*\emptyset) = \emptyset \cup \emptyset = \emptyset;$$

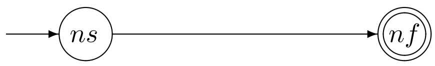
$$R'_{n,n} = R_{n,n} \cup (R_{n,s}R^*_{s,s}R_{s,n}) = a \cup (\emptyset \ldots) = a \cup \emptyset = a;$$

$$R'_{n,nf} = R_{n,nf} \cup (R_{n,s}R^*_{s,s}R_{s,nf}) = \Lambda \cup (\emptyset \ldots) = \Lambda \cup \emptyset = \Lambda.$$

Thus, the 3-state a-automaton takes the following form:



Eliminating the state n. Now, all that remains to do is to go from here to the 2-state a-automaton by eliminating the remaining state n:



The final expression is the corresponding expression for $R'_{ns,nf}$:

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,n}R^*_{n,n}R_{n,nf}) =$$

$$\emptyset \cup (Aa^*\Lambda) = Aa^*\Lambda = Aa^*.$$

Resulting answer: The regular expression corresponding to the original automaton is Aa^*.