Solution to Problem 6

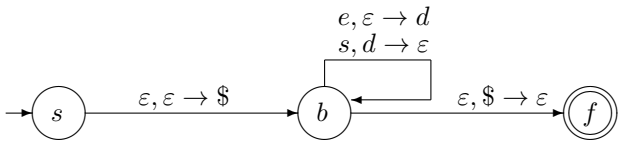
Task: Show, step by step, how the following pushdown automaton – that checks whether a word consisting of letters e and s corresponds to a balanced budget – will accept the word *eses*. This pushdown automaton has three states:

- the starting state s,
- the state b meaning that so far, we have earned at least as much as we spent, and
- the final state f.

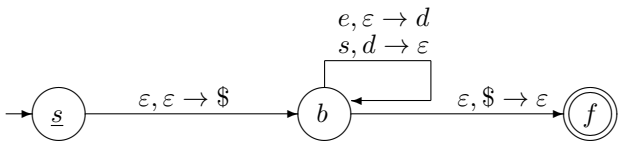
The transitions are as follows:

- From s to b, the transition is $\varepsilon, \varepsilon \to \$$;
- From b to b, the transitions are: $e, \varepsilon \to d$ and $s, d \to \varepsilon$.
- From b to f, the transition is: $\varepsilon, \$ \to \varepsilon$.

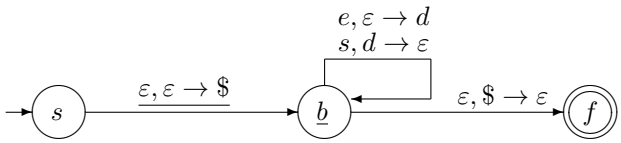
Solution. Our pushdown automaton has the following form:



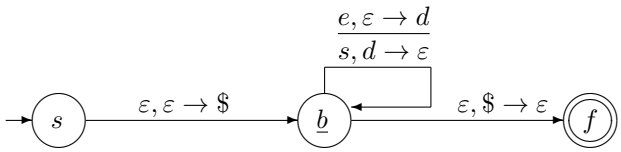
We start in the starting state s with an empty stack:



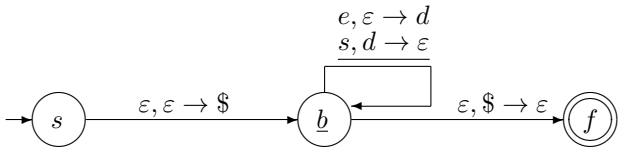
We want to eventually reach the final state, and the only way to get out of the starting state is to use the rule $\varepsilon, \varepsilon \to \$$ and go to state b:



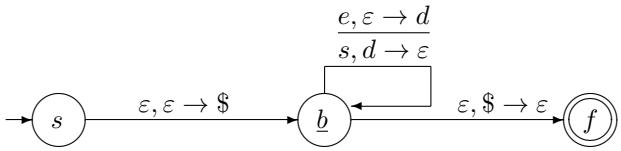
The stack now contains the dollar sign. After that, we read the first symbol e and thus, use the rule $e, \varepsilon \to d$:



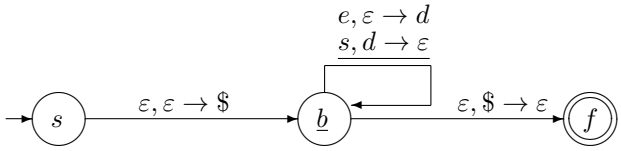
The stack now contains a symbol d on top of the dollar sign. Then, we read the next symbol s and use the rule $s, d \to \varepsilon$:



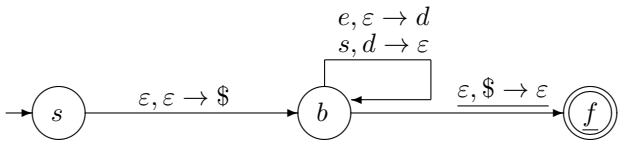
Because of this rule, the top symbol d is popped out of the stack, so we only have the dollar sign in the stack. New, we again we read the symbol e and use the rule $e, \varepsilon \to d$:



The stack now contains a symbol d on top of the dollar sign. Then, we read the last symbol s and use the rule $s, d \to \varepsilon$:



We have read all the symbols of the word. The stack now contains only one symbol – the dollar sign. So, to get to the final state, we can use the rule $\varepsilon, \$ \to \varepsilon$:



We are now in the final state with an empty stack, so the word *eses* is accepted.

To illustrate these transitions, let us list all the symbols we read, all the states that this automaton goes through, and under each state, the contents of the corresponding stack, with $\to$ indicating transition corresponding to reading a symbol:

read			e		s		e		s		
state	s	b	$\to$	b	$\to$	b	$\to$	b	$\to$	b	f
stack		\$	$\to$	d \$	$\to$	\$	$\to$	d \$	$\to$	\$	