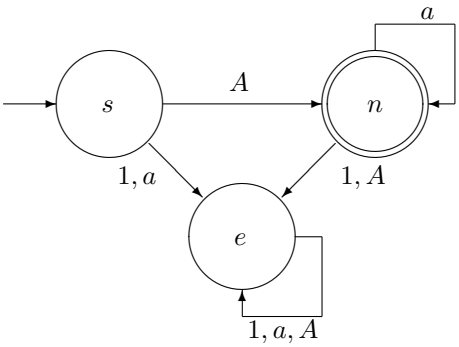
Solution to Homework 8

Tasks: In the corresponding lecture, we described an algorithm that, given a finite automaton, produces a context-free grammar – a grammar that generate a word if and only if this word is accepted by the given automaton.

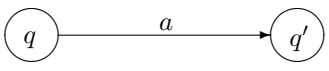
1. On the example from the automaton from Homework 1.1, show how this algorithm will generate the corresponding context-free grammar. Similarly to Homework 3, assume that we only have symbols A, a, and 1.
2. On the example of a word Aaa accepted by this automaton, show how the tracing of acceptance of this word by the finite automaton can be translated into a generation of this same word by your context-free grammar.

Reminder. The automaton has the following form:



Solution to Task 1. The general algorithm for transforming FA into CFG is as follows:

- To each state q of the FA, introduce a new variable Q.
- The variable corresponding to the starting state will be the starting variable of the new CFG.
- For each transition of the finite automaton



we add a rule $Q \to aQ'$.

- For each final state f of the FA, we add a rule $F \to \varepsilon$.

By applying this general algorithm to this FA, we get a CFG with 3 variables S, N, and E, three terminal symbols A, a, ansd 1, the starting variable S and the following rules:

$$S \to AN$$
$$S \to aE$$
$$S \to 1E$$
$$N \to AE$$
$$N \to aN$$
$$N \to 1E$$
$$E \to AE$$
$$E \to aE$$
$$E \to 1E$$
$$N \to \varepsilon$$

Solution to Task 2. Derivations in this grammar follow, step-by-step, the way the original finite automaton accepts a word. The word Aaa is accepted by the original finite automaton as follows:

- we start in the start state s; this corresponds to the starting variable S;
- then, we use the fact that once we are in the state s and we see the symbol A, then we move to the state n; this transition corresponds to the rule $S \to AN$, so the generation so far is:

$$\underline{S} \to AN;$$

- then, we use the fact that once we are in the state n and we see the symbol a, then we go back to the state n; this transition corresponds to the rule $N \to aN$, so generation so far is

$$\underline{S} \to A\underline{N} \to AaN;$$

- then, we again use the fact that once we are in the state n and we see the symbol a, then we go back to the state n; this transition corresponds to the same rule $N \to aN$, so generation so far is

$$\underline{S} \to A\underline{N} \to Aa\underline{N} \to AaaN;$$

- we have read all the symbols of the word, and we are in the final state n; for the FA, this means that the word Aaa is accepted; for CFG, we need to use the rule $N \to \varepsilon$ corresponding to the final state n; thus, we get the following derivation of the word Aaa:

$$\underline{S} \to A\underline{N} \to Aa\underline{N} \to Aaa\underline{N} \to Aaa.$$

So, we have indeed derived the word Aaa in the grammar.