Solution to Quiz 2

Task. Transform the following grammar, with the starting variable $A$, into the Chomsky normal form:

$$A \rightarrow BaB, B \rightarrow AaA, A \rightarrow \varepsilon.$$

Solution. On the preliminary step, we add a new starting variable $S_0$ and a new rule

$$S_0 \rightarrow A.$$

On Step 0, we get rid of the rule $A \rightarrow \varepsilon$ by adding, for each rule that has $A$ in the right-hand side, rules where this $A$ is deleted. Thus, to the previous rules

$$A \rightarrow BaB, B \rightarrow AaA, S_0 \rightarrow A,$$

we add rules

$$B \rightarrow aA, B \rightarrow Aa, B \rightarrow a, S_0 \rightarrow \varepsilon.$$

On Step 1, we delete the rule $S_0 \rightarrow A$, by adding, to each rule where $A$ goes to something, a similar rule in which $S + 0$ goes to that same “something”. Thus, to the previous rules

$$A \rightarrow BaB, B \rightarrow AaA, B \rightarrow aA, B \rightarrow Aa, B \rightarrow a, S_0 \rightarrow \varepsilon,$$

we add a new rule

$$S_0 \rightarrow BaB.$$

On Step 2, we add a new variable $V_a$, a new rule $V_a \rightarrow a$, and in each rule of length 2 or more that has terminal symbols in the right-hand side, we replace these terminal symbols with the corresponding variable. As a result, we get the following rules:

$$A \rightarrow BV_aB, B \rightarrow AV_aA, B \rightarrow V_aB, B \rightarrow AV_a, B \rightarrow a, S_0 \rightarrow \varepsilon, S_0 \rightarrow BV_aB,$$

$$V_a \rightarrow a.$$

On Step 3, we replace each rule of the type $A \rightarrow BCD$ with two rules $V_{BC} \rightarrow BC$ and $A \rightarrow V_{BC}D$. As a result, we get the following grammar which is already in Chomsky normal form:

$$V_{Ba} \rightarrow BV_a, A \rightarrow V_{Ba}B, V_{Aa} \rightarrow AV_a, B \rightarrow V_{Aa}A, B \rightarrow V_aA, B \rightarrow AV_a, B \rightarrow a,$$

$$S_0 \rightarrow \varepsilon, S_0 \rightarrow V_{Ba}B, V_a \rightarrow a.$$