Solution to Quiz 2

Task. Transform the following grammar, with the starting variable A, into the Chomsky normal form:

$$A \to BaB, B \to AaA, A \to \varepsilon.$$

Solution. On the preliminary step, we add a new starting variable S_0 and a new rule

$$S_0 \to A.$$

On Step 0, we get rid of the rule $A \to \varepsilon$ by adding, for each rule that has A in the right-hand side, rules where this A is deleted. Thus, to the previous rules

$$A \to BaB, B \to AaA, S_0 \to A,$$

we add rules

$$B \to aA, B \to Aa, B \to a, S_0 \to \varepsilon.$$

On Step 1, we delete the rule $S_0 \to A$, by adding, to each rule where A goes to something, a similar rule in which $S+0$ goes to that same "something". Thus, to the previous rules

$$A \to BaB, B \to AaA, B \to aA, B \to Aa, B \to a, S_0 \to \varepsilon,$$

we add a new rule

$$S_0 \to BaB.$$

On Step 2, we add a new variable V_a, a new rule $V_a \to a$, and in each rule of length 2 or more that has terminal symbols in the right-hand side, we replace these terminal symbols with the corresponding variable. As a result, we get the following rules:

$$A \to BV_aB, B \to AV_aA, B \to V_aA, B \to AV_a, B \to a, S_0 \to \varepsilon, S_0 \to BV_aB,$$
$$V_a \to a.$$

On Step 3, we replace each rule of the type $A \to BCD$ with two rules $V_{BC} \to BC$ and $A \to V_{BC}D$. As a result, we get the following grammar which is already in Chomsky normal form:

$$V_{Ba} \to BV_a, A \to V_{Ba}B, V_{Aa} \to AV_a, B \to V_{Aa}A, B \to V_aA, B \to AV_a, B \to a,$$
$$S_0 \to \varepsilon, S_0 \to V_{Ba}B, V_a \to a.$$