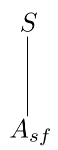
Solution to Homework 11

Task. Use the general algorithm to transform the pushdown automaton from Problem 6 into a context-free grammar. Show, step-by-step, how the resulting grammar will generate the word $IDII$.

Solution. Let us recall how the word $IDII$ is accepted by this automaton:

read			I		D	I		I				
state	s	w	a_1	w	w	a_1	w	a_2	w	f	f	f
stack		\$	\$	I \$	\$	\$	I \$	I \$	I I \$	I \$	\$	

We start with the state s, we end up in the final state f. Thus, the first rule we apply if the rule $S \to A_{sf}$;



The first symbol we push is the dollar sign, this dollar sign is popped at the end. Thus, we have the following combination of pop=push rules:

In general, we have the two transitions

What do we need to plug in instead of p, q, etc. in the general 2-rule picture to come up with this particular picture:

- instead of p, we place s;

- instead of q, we place w;
- instead of s and t, we place f;
- instead of x and y, we place ε.

If we make these substitutions in the general rule:

$$A_{ps} \to xA_{qr}y,$$

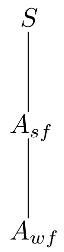
we get the rule

$$A_{sf} \to \varepsilon A_{wf}\varepsilon.$$

Since concatenation with the empty string does not change anything, this means

$$A_{sf} \to A_{wf}.$$

Thus, the derivation so far takes the following form:



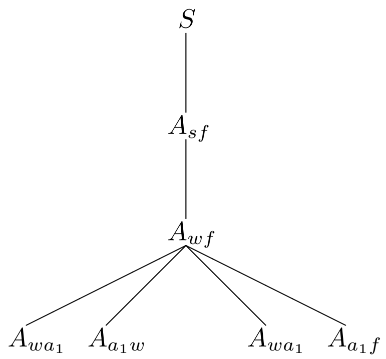
We covered how we push the dollar sign and how we pop it. Let us underline what we have covered:

read			I		D	I		I				
state	s	w	a_1	w	w	a_1	w	a_2	w	f	f	f
stack		$\underline{\$}$	$\underline{\$}$	I $\underline{\$}$	$\underline{\$}$	$\underline{\$}$	I $\underline{\$}$	I $\underline{\$}$	I I $\underline{\$}$	I $\underline{\$}$	$\underline{\$}$	

If we ignore the dollar signs – since we already took care of them – then we see that we have three intermediate states with the empty stack. So, we need to use the transitivity rule, which in this case takes the form

$$A_{wf} \to A_{wa_1}A_{a_1w}A_{wa_1}A_{a_1f}.$$

Thus, the derivation tree takes the following form:



Transition A_{wa_1}. In the first transition from w to a_1 we have only one rule, so we add a fictitious rule to form a pair. We ignore the dollar signed that do not change:

This combination leads to the rule $A_{wa_1} \to IA_{a_1a_1}\varepsilon$, i.e., $A_{wa_1} \to IA_{a_1a_1}$. Here, the remaining transition between a_1 and a_1 does not include any additional steps, so we can use the rule $A_{a_1a_1} \to \varepsilon$. So, we get the rule $A_{wa_1} \to I$.

Transition A_{a_1w}. For the transition between a_1 and w, we push I and then pop I:

So, we have a transition $A_{a_1w} \to \varepsilon A_{ww}D$, i.e., $A_{a_1w} \to A_{ww}D$. Here, the remaining transition between w and w does not include any additional steps, so we can use the rule $A_{ww} \to \varepsilon$. So, we get the rule $A_{a_1w} \to D$.

Transition A_{wa_1}. The next transition A_{wa_1} is the same as the first transition from w to a_1,before, so it corresponds to the rule $A_{wa_1} \to I$.

Transition A_{a_1f}. In the last transition A_{a_1f}, we first push I, then pop I:

So, we get the transition $A_{a_1f} \to \varepsilon A_{wf}\varepsilon$, i.e., $A_{a_1f} \to A_{wf}$. This takes care of the second I, so we get:

read			I		D	I		I				
state	s	w	a_1	w	w	a_1	w	a_2	w	f	f	f
stack		$\underline{\$}$	$\underline{\$}$	$\underline{I}$ $\$$	$\underline{\$}$	$\underline{\$}$	$\underline{I}$ $\$$	$\underline{I}$ $\$$	I $\underline{I}$ $\$$	$\underline{I}$ $\$$	$\underline{\$}$	

Now, if we ignore the second I, we get an intermediate state a_2 with an empty stack, so we need to use transitivity $A_{wf} \to A_{wa_2}A_{a_2f}$. For A_{wa_2}, there is only one rule, so we pair it with a fictitious trivial rule:

So, we get $A_{wa_2} \to IA_{a_2a_2}\varepsilon$, i.e., $A_{wa_2} \to IA_{a_2a_2}$. There are no rules for the transition $A_{a_2a_2}$, so we get $A_{a_2a_2} \to \varepsilon$ and thus, $A_{wa_2} \to I$.

Finally, for the transition A_{a_2f}, we push and pop the last I by using the following rules:

Here, we get the rule $A_{a_2f} \to \varepsilon A_{ww}\varepsilon$, i.e., $A_{a_2f} \to A_{ww}$. There are no rules for the transition A_{ww}, so we get $A_{ww} \to \varepsilon$ and thus, $A_{wa_2} \to \varepsilon$. So, the transition takes the following form: