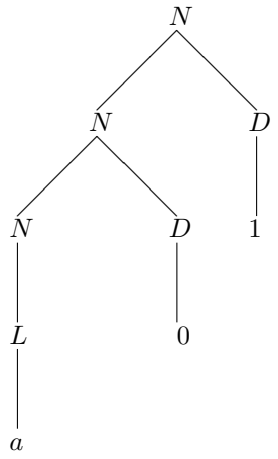
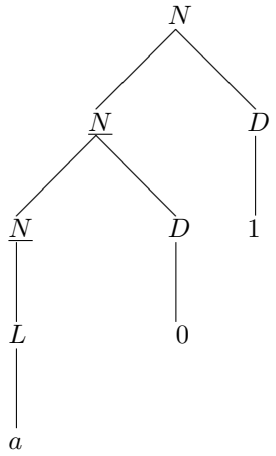
Solution to Problem 12

Task. For the grammar described in Homework 7, show how the word $a01$ can be represented as $uvxyz$ in accordance with the pumping lemma for context-free grammars. Show that the corresponding word $uvvxyyz$ will be generated by this grammar.

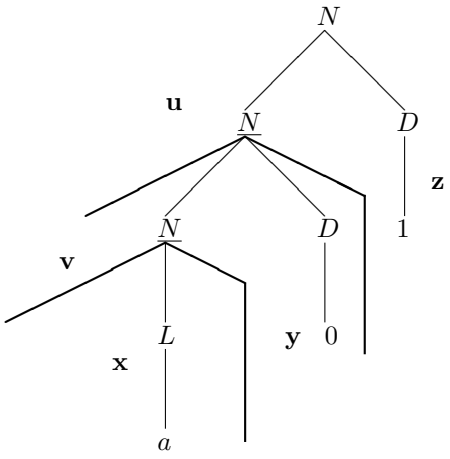
Solution. The derivation of this string takes the following form:



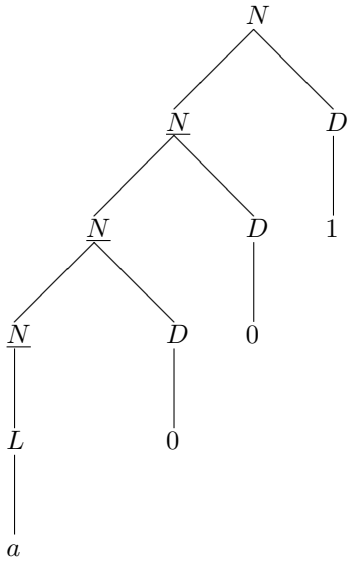
The lowest pair of occurrences of the same variable is the lowest pair of occurrences of the variable N:



Thus, the desired decomposition of this word into u, v, x, y, and z has the following form:



So, here $u = v = \varepsilon$, $x = a$, $y = 0$, and $z = 1$. If we copy of the part between the two lowest occurrences of N to the lower occurrence, we conclude that the word $uvvxyyz$ can be derived as follows:


N
N
D
N
D
1
N
D
0
L
0
a