Solution to Homework Problem 15

Task. Design a Turing machine that, given a unary number n, adds 2 to this number. Test it, step-by-step, on the example of $n = 3$.

Idea. This is similar to adding 1 to a unary number. The only difference is that after we added 1, we need to add one more 1. Then, we go back.

Solution. Here are the rules for the Turing machine:
start, – → R, moving
moving, 1 → R
moving, – → 1, added1st1, R
added1st1, – → 1, back, L
back, 1 → L
back, – → halt

Tracing.

<u>–</u>	1	1	1	–	–	–	...	start
–	<u>1</u>	1	1	–	–	–	...	working
–	1	<u>1</u>	1	–	–	–	...	working
–	1	1	<u>1</u>	–	–	–	...	working
–	1	1	1	<u>–</u>	–	–	...	working
–	1	1	1	1	<u>–</u>	–	...	added1st1
–	1	1	1	<u>1</u>	1	–	...	back
–	1	1	<u>1</u>	1	1	–	...	back
–	1	<u>1</u>	1	1	1	–	...	back
–	<u>1</u>	1	1	1	1	–	...	back
<u>–</u>	1	1	1	1	1	–	...	back
<u>–</u>	1	1	1	1	1	–	...	halt