Solution to Homework Problem 16

Task. Design a Turing machine that, given a positive binary number n greater than or equal to 4, subtracts 4 from this number. Test it, step-by-step, on the example of $n = 6$.

Solution. Here are the rules for the Turing machine:
start, $-$ $\to$ R, skip1st
skip1st, 0 $\to$ R, skip2nd
skip1st, 1 $\to$ R, skip2nd
skip2nd, 0 $\to$ R, moving
skip2nd, 1 $\to$ R, moving
moving, 0 $\to$ 1, R, moving
moving, 1 $\to$ 0, L, back
back, 1 $\to$ L
back, 0 $\to$ L
back, $-$ $\to$ halt

Tracing. We start with the number $6_{10} = 110_2$ which is represented as 011.

$\underline{-}$	0	1	1	$-$	$-$	$-$	...	start

$-$	$\underline{0}$	1	1	$-$	$-$	$-$	...	skip1st

$-$	0	$\underline{1}$	1	$-$	$-$	$-$	...	skip2nd

$-$	0	1	$\underline{1}$	$-$	$-$	$-$	...	moving

$-$	0	$\underline{1}$	0	$-$	$-$	$-$	...	back

$-$	$\underline{0}$	1	0	$-$	$-$	$-$	...	back

$\underline{-}$	0	1	0	$-$	$-$	$-$	...	back

$\underline{-}$	0	1	0	$-$	$-$	$-$	...	halt