Task 2.1. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language $(a \cup b)^* b$ of all the words that contain only letters $a$ and $b$ and that end in $b$.

Reminder:

- $a$ and $b$ are languages consisting of only one 1-symbol word each: $a$ is a language consisting of a single 1-symbol word $a$; $b$ is a language consisting of a single 1-symbol word $b$;
- for any two languages $C$ and $D$, the notation $CD$ means concatenation.

Solution. We start with the standard non-deterministic automata for recognizing:

- the language $a$ – that consists of a single word $a$, and
- the language $b$ – that consists of a single word $b$:

![Diagram 1](image)

Then, we use the general algorithm for the union to design a non-deterministic automaton for recognizing the language $a \cup b$:

![Diagram 2](image)
Now, we apply a standard algorithm for the Kleene star, and we get the following non-deterministic automaton for $(a \cup b)^*$:

Now, we use the algorithm for concatenation for combine them: final states of the automaton for $A$ are no longer final, and from each of them, we add a jump to the starting state of the automaton for $(a \cup b)^*b$:
Task 2.2. Transform the resulting non-deterministic finite automaton into a deterministic one.

Solution. Let us first enumerate the states of the resulting non-deterministic automaton.

In the beginning, before we see any symbol, we are in state 3, and we can jump to 1, 4, 5, and 7. So, the resulting state is \(\{1, 3, 4, 5, 7\}\).

- If in the state \(\{1, 3, 4, 5, 7\}\), we see letter \(a\), we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is \(\{1, 2, 4, 5, 7\}\).

- If in the state \(\{1, 3, 4, 5, 7\}\), we see letter \(b\), we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is \(\{1, 4, 5, 6, 7, 8\}\). This state contains the final state 6 and is, thus, final.

- If in the state \(\{1, 2, 4, 5, 7\}\), we see letter \(a\), we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is the same state \(\{1, 2, 4, 5, 7\}\).

- If in the state \(\{1, 2, 4, 5, 7\}\), we see letter \(b\), we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is \(\{1, 4, 5, 6, 7, 8\}\).

- If in the state \(\{1, 4, 5, 6, 7, 8\}\), we see letter \(a\), we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is \(\{1, 2, 4, 5, 7\}\).

- If in the state \(\{1, 4, 5, 6, 7, 8\}\), we see letter \(b\), we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is the same state \(\{1, 4, 5, 6, 7, 8\}\).

Thus, we arrive at the following deterministic automaton.