Task 2.1. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language \((a \cup b)^* b\) of all the words that contain only letters \(a\) and \(b\) and that end in \(b\).

Reminder:

- \(a\) and \(b\) are languages consisting of only one 1-symbol word each: \(a\) is a language consisting of a single 1-symbol word \(a\); \(b\) is a language consisting of a single 1-symbol word \(b\);
- for any two languages \(C\) and \(D\), the notation \(CD\) means concatenation.

Solution. We start with the standard non-deterministic automata for recognizing:

- the language \(a\) – that consists of a single word \(a\), and
- the language \(b\) – that consists of a single word \(b\):

Then, we use the general algorithm for the union to design a non-deterministic automaton for recognizing the language \(a \cup b\):
Now, we apply a standard algorithm for the Kleene star, and we get the following non-deterministic automaton for \((a \cup b)^*\): 

Now, we use the algorithm for concatenation for combine them: final states of the automaton for \(A\) are no longer final, and from each of them, we add a jump to the starting state of the automaton for \((a \cup b)^*b\):
Task 2.2. Transform the resulting non-deterministic finite automaton into a deterministic one.

Solution. Let us first enumerate the states of the resulting non-deterministic automaton.

In the beginning, before we see any symbol, we are in state 3, and we can jump to 1, 4, 5, and 7. So, the resulting state is \{1, 3, 4, 5, 7\}.

- If in the state \{1, 3, 4, 5, 7\}, we see letter a, we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is \{1, 2, 4, 5, 7\}.

- If in the state \{1, 3, 4, 5, 7\}, we see letter b, we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is \{1, 4, 5, 6, 7, 8\}. This state contains the final state 6 and is, thus, final.

- If in the state \{1, 2, 4, 5, 7\}, we see letter a, we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is the same state \{1, 2, 4, 5, 7\}.

- If in the state \{1, 2, 4, 5, 7\}, we see letter b, we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is \{1, 4, 5, 6, 7, 8\}.

- If in the state \{1, 4, 5, 6, 7, 8\}, we see letter a, we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is \{1, 2, 4, 5, 7\}.

- If in the state \{1, 4, 5, 6, 7, 8\}, we see letter b, we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is the same state \{1, 4, 5, 6, 7, 8\}.

Thus, we arrive at the following deterministic automaton.