Solutions to Homework 2

Task 2.1. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language $(a \cup b)^* b$ of all the words that contain only letters a and b and that end in b.

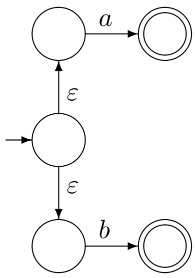
Reminder:

- a and b are languages consisting of only one 1-symbol word each: a is a language consisting of a single 1-symbol word a; b is a language consisting of a single 1-symbol word b;
- for any two languages C and D, the notation CD means concatenation.

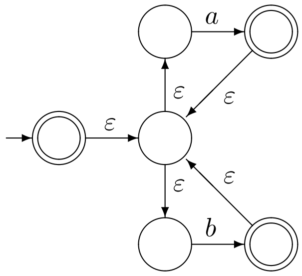
Solution. We start with the standard non-deterministic automata for recognizing:

- the language a – that consists of a single word a, and
- the language b – that consists of a single word b:

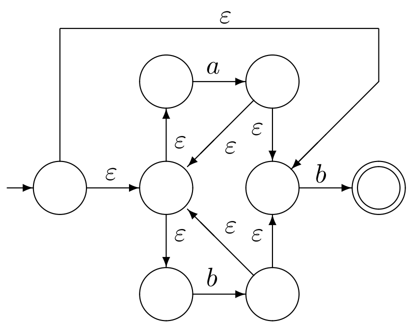
Then, we use the general algorithm for the union to design a non-deterministic automaton for recognizing the language $a \cup b$:



Now, we apply a standard algorithm for the Kleene star, and we get the following non-deterministic automaton for $(a \cup b)^*$:

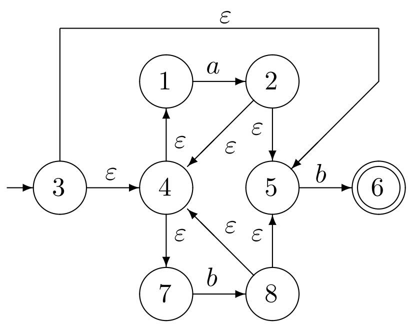


Now, we use the algorithm for concatenation for combine them: final states of the automaton for A are no longer final, and from each of them, we add a jump to the starting state of the automaton for $(a \cup b)^*b$:



Task 2.2. Transform the resulting non-deterministic finite automaton into a deterministic one.

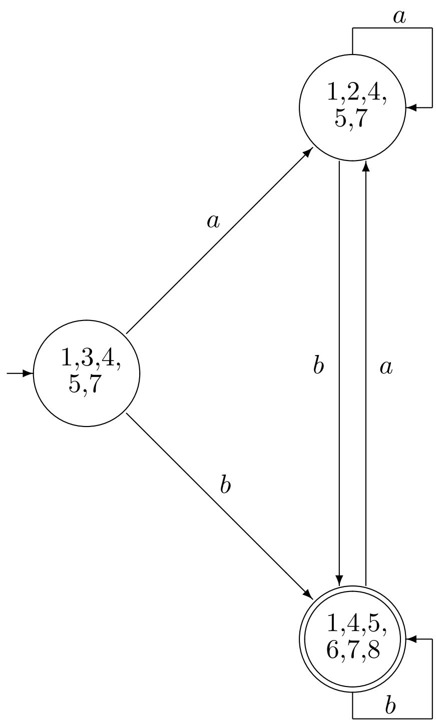
Solution. Let us first enumerate the states of the resulting non-deterministic automaton.



In the beginning, before we see any symbol, we are in state 3, and we can jump to 1, 4, 5, and 7. So, the resulting state is $\{1, 3, 4, 5, 7\}$.

- If in the state $\{1, 3, 4, 5, 7\}$, we see letter a, we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is $\{1, 2, 4, 5, 7\}$.
- If in the state $\{1, 3, 4, 5, 7\}$, we see letter b, we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is $\{1, 4, 5, 6, 7, 8\}$. This state contains the final state 6 and is, thus, final.
- If in the state $\{1, 2, 4, 5, 7\}$, we see letter a, we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is the same state $\{1, 2, 4, 5, 7\}$.
- If in the state $\{1, 2, 4, 5, 7\}$, we see letter b, we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is $\{1, 4, 5, 6, 7, 8\}$.
- If in the state $\{1, 4, 5, 6, 7, 8\}$, we see letter a, we can go to 2 and from there, jump to 1, 4, 5, and 7. Thus, the resulting state is $\{1, 2, 4, 5, 7\}$.
- If in the state $\{1, 4, 5, 6, 7, 8\}$, we see letter b, we can go to 8 and from there, jump to 1, 4, 5, and 7. We can also go to 6. Thus, the resulting state is the same state $\{1, 4, 5, 6, 7, 8\}$.

Thus, we arrive at the following deterministic automaton.



a
1,2,4,
5,7
a
1,3,4,
5,7
b
a
b
1,4,5,
6,7,8
b