

## Solution to Homework Problem 15

**Task.** Design a Turing machine that, given a unary number  $n$  which is great than or equal to 1, subtracts 2 from this number. Test it, step-by-step, on the example of  $n = 2$ .

**Idea.** This is similar to subtracting 1 to a unary number. The only difference is that after we subtract 1, we need to subtract one more 1. Then, we go back.

**Solution.** Here are the rules for the Turing machine:

start,  $- \rightarrow R$ , moving  
 moving,  $1 \rightarrow R$   
 moving,  $- \rightarrow L$ , deleteFirst, L  
 deleteFirst,  $1 \rightarrow -$ , deleteSecond, L  
 deleteSecond,  $1 \rightarrow -$ , back, L  
 back,  $1 \rightarrow L$   
 back,  $- \rightarrow \text{halt}$

**Tracing.**

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-	1	1	-	-	-	-	...		
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-	<u>1</u>	1	-	-	-	-	...		
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-	1	<u>1</u>	-	-	-	-	...		
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-	1	1	-	-	-	-	...		
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-	1	<u>1</u>	-	-	-	-	...		
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-	<u>1</u>	-	-	-	-	-	...		
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-	-	-	-	-	-	-	...		
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