# Solution to Homework Problem 16

**Task.** Design a Turing machine that, given a binary number $n$, adds 4 to this number. Test it, step-by-step, on the example of $n = 1$.

**Solution.** Here are the rules for the Turing machine:
start, – → R, skip1st
skip1st, 0 → R, skip2nd
skip1st, 1 → R, skip2nd
skip1st, – → 0, R, skip2nd
skip2nd, 0 → R, moving
skip2nd, 1 → R, moving
skip2nd, – → 0, R, moving
moving, 1 → 0, R, moving
moving, 0 → 1, L, back
moving, – → 1, L, back
back, 1 → L
back, 0 → L
back, – → halt

**Tracing.** We start with the number $1_{10} = 1_2$ which is represented as 011.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\underline{-}$ | 1 | – | – | – | – | – | ... | start |
| – | $\underline{1}$ | – | – | – | – | – | ... | skip1st |
| – | 1 | $\underline{-}$ | – | – | – | – | ... | skip2nd |
| – | 1 | 0 | $\underline{-}$ | – | – | – | ... | moving |
| – | 1 | $\underline{0}$ | 1 | – | – | – | ... | back |
| – | $\underline{1}$ | 0 | 1 | – | – | – | ... | back |
| $\underline{-}$ | 1 | 0 | 1 | – | – | – | ... | back |
| $\underline{-}$ | 1 | 0 | 1 | – | – | – | ... | halt |