Solution to Problem 6

Task: Show, step by step, how the following pushdown automaton – that checks whether a word consisting of letters A and B corresponds to a student with GPA higher than 3.5 – will accept the word ABAA. This pushdown automaton has three states:

- the starting state s,
- the working state w, and
- the final state f.

In the stack, in addition to the bottom symbol \$, we have:

- either one or several As – indicating how many more As than Bs the student has,
- or one or several Bs – indicating how many more Bs than As the student has.

The transitions are as follows:

- From s to w, the transition is $\varepsilon, \varepsilon \to \$$.
- From w to f, the transition is: $\varepsilon, \mathrm{A} \to \varepsilon$.
- From f to f, we have two transitions: $\varepsilon, \mathrm{A} \to \varepsilon$ and $\varepsilon, \$ \to \varepsilon$.

From w to w, we have the following transitions:

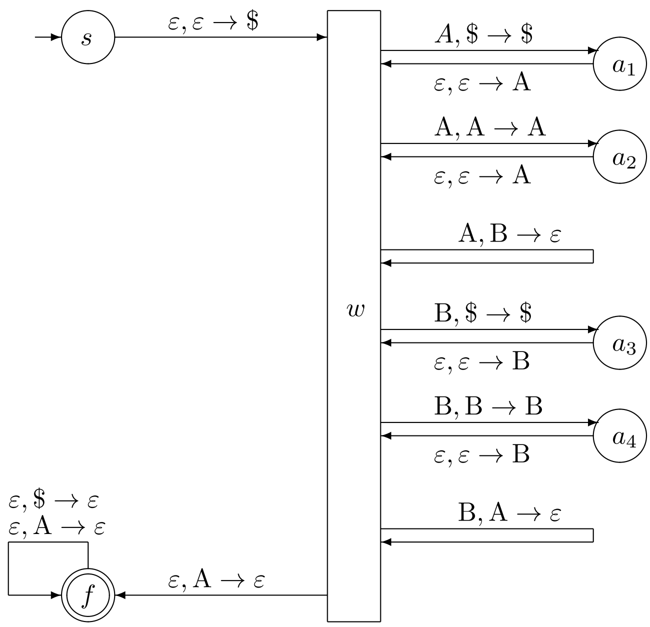
- If we see the symbol A and \$ is on top of the stack, we keep the dollar sign and add A to the stack, i.e., we have transition $\mathrm{A}, \$ \to \$$ that brings us to an intermediate state a_1, and then the transition $\varepsilon, \varepsilon \to \mathrm{A}$ that brings us back to the working state.
- If we see the symbol A and A is on top of the stack, we keep the top A and add another A to the stack, i.e., we have transition $\mathrm{A}, \mathrm{A} \to \mathrm{A}$ that brings us to an intermediate state a_2, and then the transition $\varepsilon, \varepsilon \to \mathrm{A}$ that brings us back to the working state.
- If we see the symbol A and B is on top of the stack, we delete the top B, i.e., we have transition $\mathrm{A}, \mathrm{B} \to \varepsilon$.

- If we see the symbol B and \$ is on top of the stack, we keep the dollar sign and add B to the stack, i.e., we have transition $\mathrm{B}, \$ \to \$$ that brings us to an intermediate state a_3, and then the transition $\varepsilon, \varepsilon \to \mathrm{B}$ that brings us back to the working state.

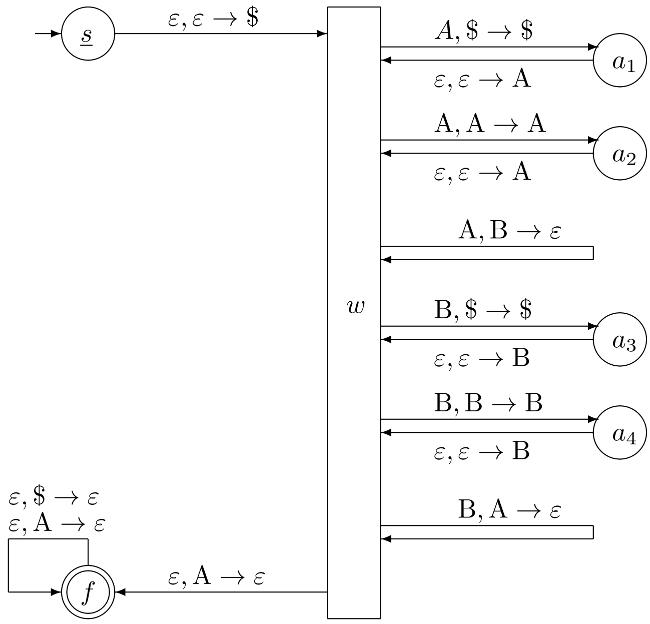
- If we see the symbol B and B is on top of the stack, we keep the top B and add another B to the stack, i.e., we have transition $\mathrm{B}, \mathrm{B} \to \mathrm{B}$ that brings us to an intermediate state a_4, and then the transition $\varepsilon, \varepsilon \to \mathrm{B}$ that brings us back to the working state.

- If we see the symbol B and A is on top of the stack, we delete the top A from the stack, i.e., we have transition $\mathrm{B}, \mathrm{A} \to \varepsilon$.

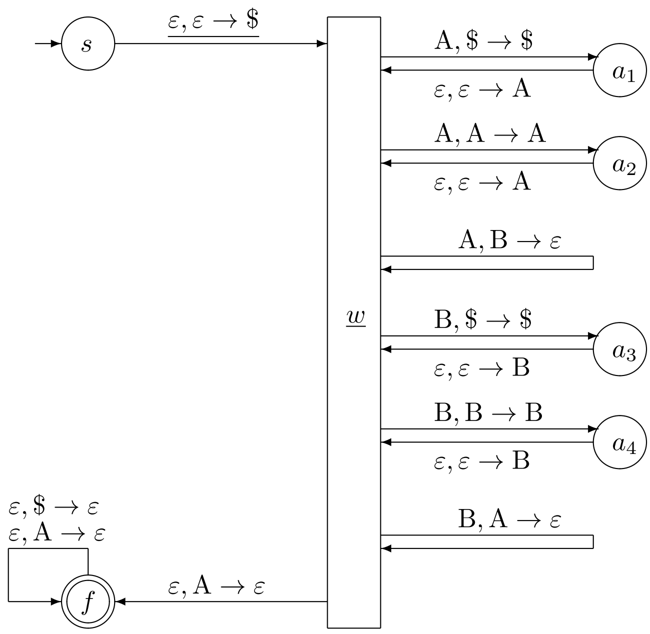
Solution. Our pushdown automaton has the following form:



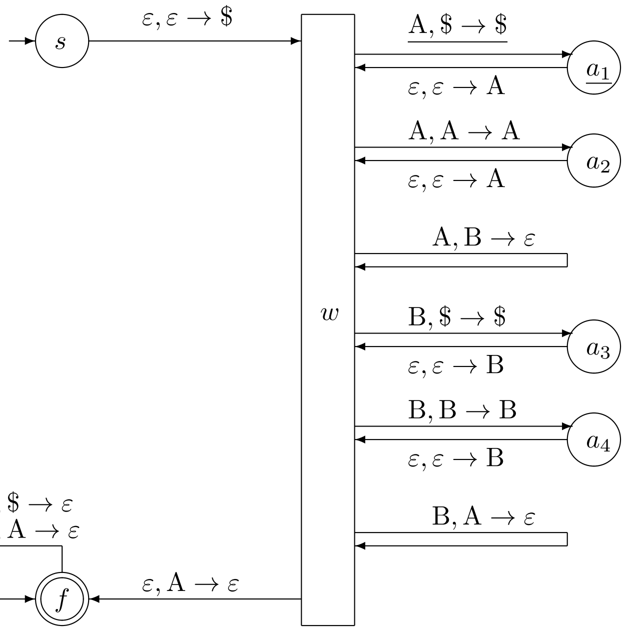
We start in the starting state s with an empty stack:



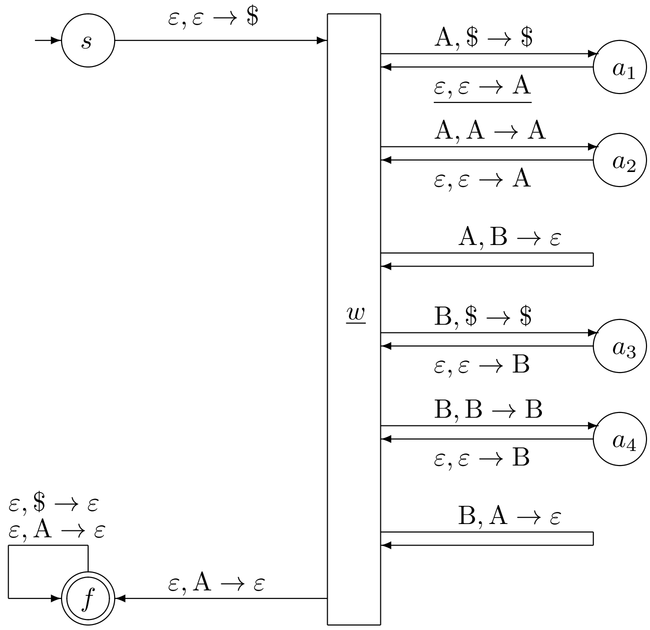
First, we add dollar sign to the stack and go to the working state:



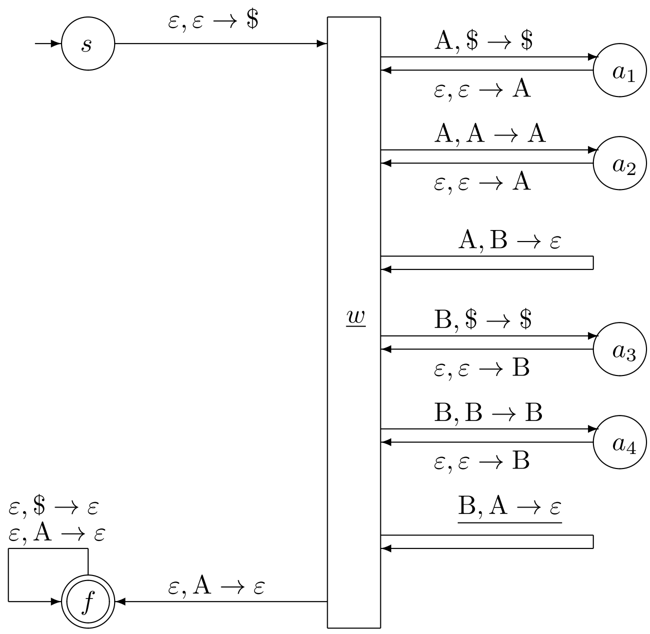
Then, we see symbol A and first go to a_1, then push A into the stack and get back to the working state:



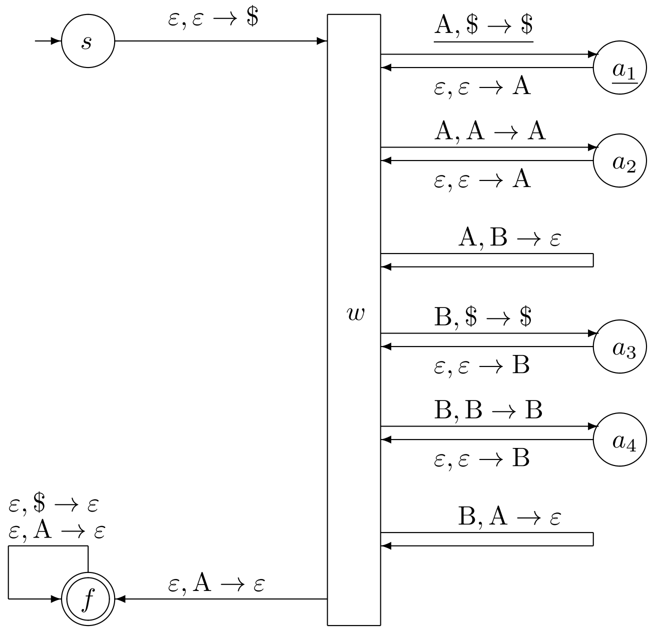
and


$\varepsilon, \varepsilon \to \$$
s
w
$\mathrm{A}, \$ \to \$$
$\varepsilon, \varepsilon \to \mathrm{A}$
a_1
$\mathrm{A}, \mathrm{A} \to \mathrm{A}$
$\varepsilon, \varepsilon \to \mathrm{A}$
a_2
$\mathrm{A}, \mathrm{B} \to \varepsilon$
$\mathrm{B}, \$ \to \$$
$\varepsilon, \varepsilon \to \mathrm{B}$
a_3
$\mathrm{B}, \mathrm{B} \to \mathrm{B}$
$\varepsilon, \varepsilon \to \mathrm{B}$
a_4
$\mathrm{B}, \mathrm{A} \to \varepsilon$
$\varepsilon, \$ \to \varepsilon$
$\varepsilon, \mathrm{A} \to \varepsilon$
f
$\varepsilon, \mathrm{A} \to \varepsilon$

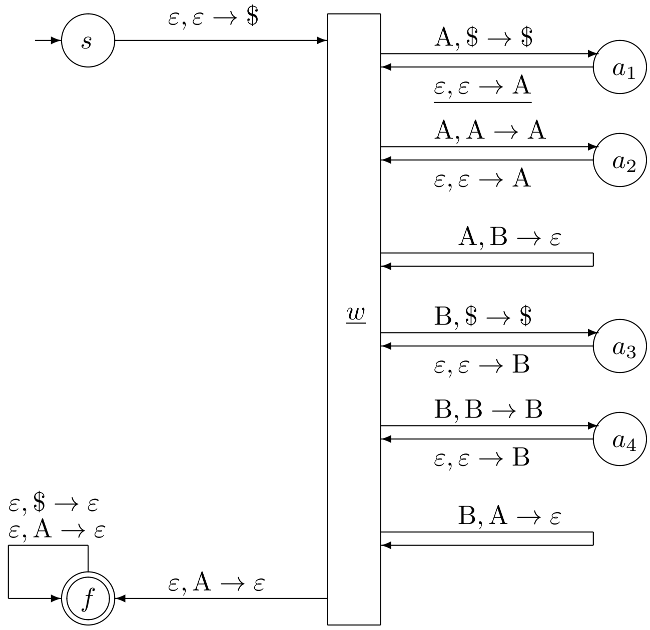
Then, we see the symbol B and delete the top A from the stack:



The stack now has only the dollar sign. Then, we read another symbol A, and again first go to a_1, then add A to the stack and go back to the working state:

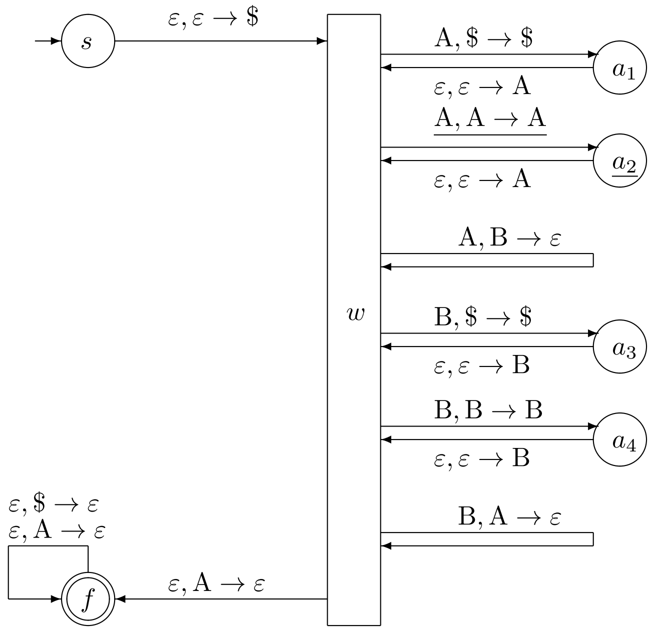


and

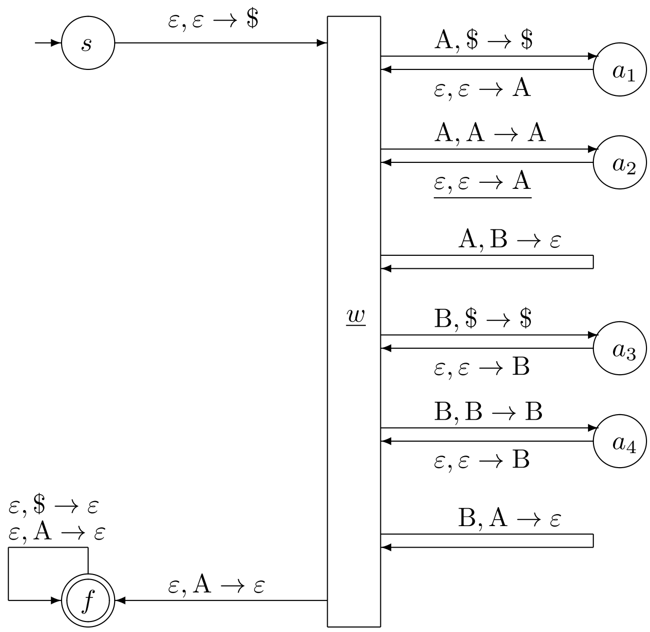


$\varepsilon, \varepsilon \to \$$
s
$\underline{w}$
$A, \$ \to \$$
a_1
$\varepsilon, \varepsilon \to A$
$A, A \to A$
a_2
$\varepsilon, \varepsilon \to A$
$A, B \to \varepsilon$
$B, \$ \to \$$
a_3
$\varepsilon, \varepsilon \to B$
$B, B \to B$
a_4
$\varepsilon, \varepsilon \to B$
$B, A \to \varepsilon$
$\varepsilon, \$ \to \varepsilon$
$\varepsilon, A \to \varepsilon$
f
$\varepsilon, A \to \varepsilon$

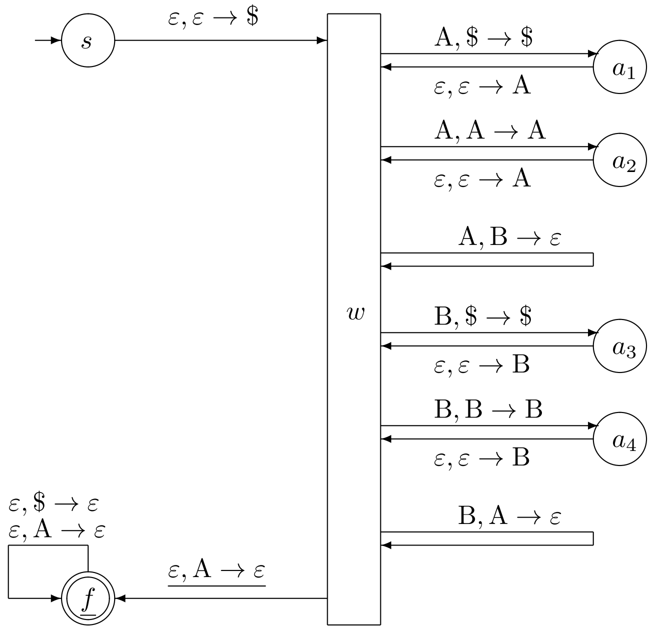
After that, we read the last symbol A. Since A is on top of the stack, we first go to a_2, then push A into the stack and go back to the working state.



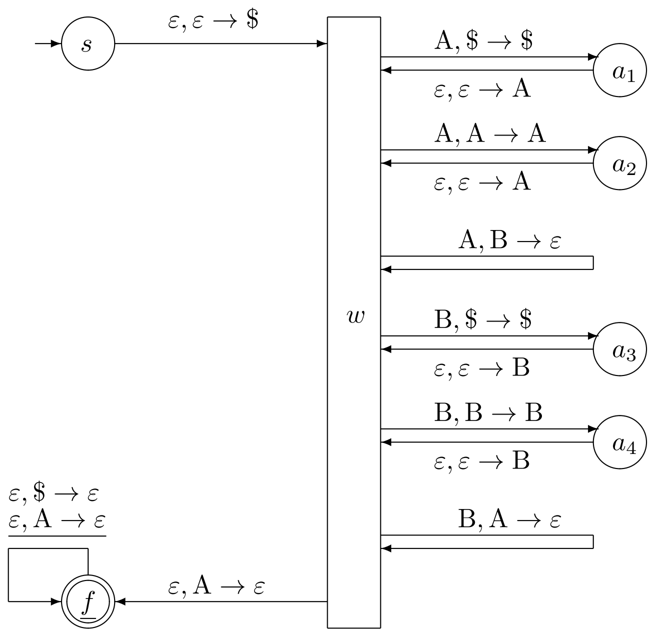
and


$\varepsilon, \varepsilon \to \$$
s
$\underline{w}$
$\mathrm{A}, \$ \to \$$
$\varepsilon, \varepsilon \to \mathrm{A}$
a_1
$\mathrm{A}, \mathrm{A} \to \mathrm{A}$
$\varepsilon, \varepsilon \to \mathrm{A}$
a_2
$\mathrm{A}, \mathrm{B} \to \varepsilon$
$\mathrm{B}, \$ \to \$$
$\varepsilon, \varepsilon \to \mathrm{B}$
a_3
$\mathrm{B}, \mathrm{B} \to \mathrm{B}$
$\varepsilon, \varepsilon \to \mathrm{B}$
a_4
$\mathrm{B}, \mathrm{A} \to \varepsilon$
$\varepsilon, \$ \to \varepsilon$
$\varepsilon, \mathrm{A} \to \varepsilon$
f
$\varepsilon, \mathrm{A} \to \varepsilon$

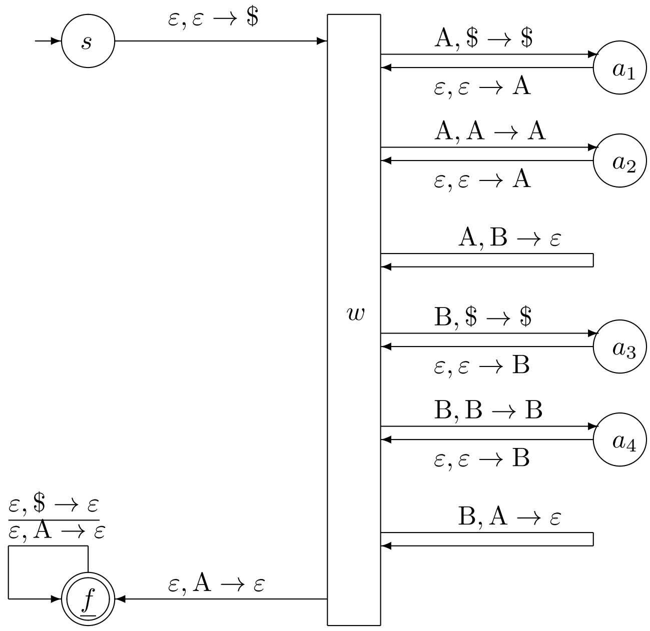
We have read all the symbols, so we move to the final state; in that move, we pop the top A from the stack, so the stack only contains one A on top of the dollar sign:



Then, we pop the symbol A from the stack, while remaining in the final state:



Finally, we pop the dollar sign from the stack, so we are now in the final state with an empty stack, so the word is accepted.



To illustrate these transitions, let us list all the symbols we read, all the states that this automaton goes through, and under each state, the contents of the corresponding stack:

read			A		B	A		A				
state	s	w	a_1	w	w	a_1	w	a_2	w	f	f	f
stack		\$	\$	A \$	\$	\$	A \$	A \$	A A \$	A \$	\$	