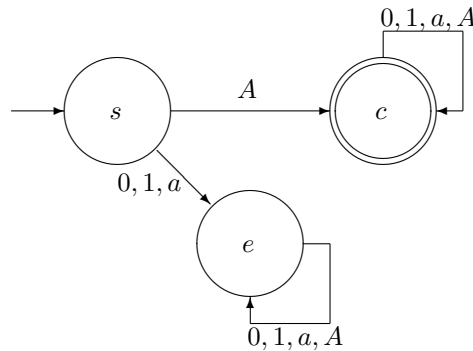


Solution to Homework 8

Tasks: In the corresponding lecture, we described an algorithm that, given a finite automaton, produces a context-free grammar – a grammar that generate a word if and only if this word is accepted by the given automaton.

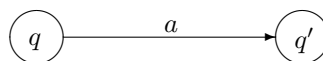
1. On the example from the automaton B from Homework 1.4, show how this algorithm will generate the corresponding context-free grammar. Similarly to Homework 3, assume that we only have symbols $0, 1, a$, and A .
2. On the example of a word $A1a$ accepted by this automaton, show how the tracing of acceptance of this word by the finite automaton can be translated into a generation of this same word by your context-free grammar.

Reminder. The automaton has the following form:



Solution to Task 1. The general algorithm for transforming FA into CFG is as follows:

- To each state q of the FA, introduce a new variable Q .
- The variable corresponding to the starting state will be the starting variable of the new CFG.
- For each transition of the finite automaton



we add a rule $Q \rightarrow aQ'$.

- For each final state f of the FA, we add a rule $F \rightarrow \varepsilon$.

By applying this general algorithm to this FA, we get a CFG with 3 variables S , C , and E , three terminal symbols A , a , and 1 , the starting variable S and the following rules:

$$S \rightarrow AC$$

$$S \rightarrow aE$$

$$S \rightarrow 0E$$

$$S \rightarrow 1E$$

$$C \rightarrow AC$$

$$C \rightarrow aC$$

$$C \rightarrow 0C$$

$$C \rightarrow 1C$$

$$E \rightarrow AE$$

$$E \rightarrow aE$$

$$E \rightarrow 0E$$

$$E \rightarrow 1E$$

$$C \rightarrow \varepsilon$$

Solution to Task 2. Derivations in this grammar follow, step-by-step, the way the original finite automaton accepts a word. The word $A1a$ is accepted by the original finite automaton as follows:

- we start in the start state s ; this corresponds to the starting variable S ;
- then, we use the fact that once we are in the state s and we see the symbol A , then we move to the state c ; this transition corresponds to the rule $S \rightarrow AC$, so the generation so far is:

$$\underline{S} \rightarrow AC;$$

- then, we use the fact that once we are in the state c and we see the symbol 1 , then we go back to the state c ; this transition corresponds to the rule $C \rightarrow 1C$, so generation so far is

$$\underline{S} \rightarrow \underline{AC} \rightarrow A1C;$$

- then, we use the fact that once we are in the state c and we see the symbol a , then we go back to the state c ; this transition corresponds to the same rule $C \rightarrow aC$, so generation so far is

$$\underline{S} \rightarrow A\underline{C} \rightarrow A1\underline{C} \rightarrow A1a\underline{C};$$

- we have read all the symbols of the word, and we are in the final state c ; for the FA, this means that the word $A1a$ is accepted; for CFG, we need to use the rule $C \rightarrow \varepsilon$ corresponding to the final state c ; thus, we get the following derivation of the word $A1a$:

$$\underline{S} \rightarrow A\underline{C} \rightarrow A1\underline{C} \rightarrow A1a\underline{C} \rightarrow A1a.$$

So, we have indeed derived the word $A1a$ in the grammar.