Automata, Computability, and Formal Languages
Fall 2023, Test 3

1-2. A perfect grading is when half of the students get Bs, one quarter As, and one quarter Cs. Let L be the language of all the sequences of letters A, B, and C that correspond to perfect grading. For example, $BABC \in L$ but $ABC \notin L$. Prove that this language is not context-free.

3. The following Turing machine deletes a binary number:

- start, – → moving, R (here, – means blank)
- moving, 0 → R
- moving, 1 → R
- moving, – → deleting, L
- deleting, 0 → –, L
- deleting, 1 → –, L
- deleting, – → halt

Trace it on the example of the word 10. Explain how each step will be represented if we interpret the Turing machine as a finite automaton with two stacks.

4. Arithmetic operations on Turing machines:

   a Design a Turing machine that subtracts 2 from a binary number.

   b Trace your Turing machine, step-by-step, on the example of the number 3.

   c Why in Turing machines (and in most actual computers) the representation of a binary number starts with the least significant digit?

5. The following finite automaton describes binary strings that start with 1:

- the starting state s;
- the final state f meaning that the first symbol was 1; and
- the error state e meaning that the first symbol was not 1.

Transitions are as follows:

- from the state s, symbol 1 leads to the state f and symbol 0 leads to the state e;
- from the state f, each symbol leads back to f;
- from the state e, each symbol leads back to e.

Use the general algorithm to transform this finite automaton into a Turing machine. Show, step-by-step, how your Turing machine will accept the string 10.

6. Give the formal definition of a feasible algorithm, and an explanation of what practically feasible means. Give two examples different from what we had in class:

- an example of a computation time which is formally feasible, but not practically feasible, and
- an example of a computation time which is practically feasible but not formally feasible.

7. What is P? What is NP? What does it means for a problem to be NP-hard? NP-complete? Give brief definitions. Give an example of an NP-complete problem: explain what is the input, what is the desired output. Is P equal to NP?