

## Solution to Homework Problem 16

**Task.** Design a Turing machine that, given a binary number  $n$ , adds 8 to this number. Test it, step-by-step, on the example of  $n = 1$ .

**Solution.** Here are the rules for the Turing machine:

start,  $- \rightarrow R$ , skip1st  
 skip1st,  $0 \rightarrow R$ , skip2nd  
 skip1st,  $1 \rightarrow R$ , skip2nd  
 skip1st,  $- \rightarrow 0$ ,  $R$ , skip2nd  
 skip2nd,  $0 \rightarrow R$ , skip3rd  
 skip2nd,  $1 \rightarrow R$ , skip3rd  
 skip2nd,  $- \rightarrow 0$ ,  $R$ , skip3rd  
 skip3rd,  $0 \rightarrow R$ , moving  
 skip3rd,  $1 \rightarrow R$ , moving  
 skip3rd,  $- \rightarrow 0$ ,  $R$ , moving  
 moving,  $1 \rightarrow 0$ ,  $R$ , moving  
 moving,  $0 \rightarrow 1$ ,  $L$ , back  
 moving,  $- \rightarrow 1$ ,  $L$ , back  
 back,  $1 \rightarrow L$   
 back,  $0 \rightarrow L$   
 back,  $- \rightarrow \text{halt}$

**Tracing.** We start with the number  $1_{10} = 1_2$  which is represented as 1.

_	1	-	-	-	-	-	...	start
-	<u>1</u>	-	-	-	-	-	...	skip1st
-	1	_	-	-	-	-	...	skip2nd
-	1	0	_	-	-	-	...	skip3rd
-	1	0	0	_	-	-	...	moving
-	1	0	<u>0</u>	1	-	-	...	back
-	1	<u>0</u>	0	1	-	-	...	back
_	1	0	0	1	-	-	...	back
_	1	0	0	1	-	-	...	halt