Solution to Homework Problem 17

**Task.** Use the general algorithm to transform a finite automaton B from Homework 1.4 – as simplified in Homework 3, into a Turing machine. Show step-by-step, on an example of a word $Aaa$, how this word will be processed by your Turing machine.

**Automaton B from Homework 1.4 as simplified in Homework 3: reminder.** This automaton has three states: $s$, $n$, and $r$; $s$ is the starting state, $n$ is the only final state. The transitions are as follows:

- from the state $s$, symbol $r$ leads to $r$, every other symbol leads to $n$;
- from the state $n$, symbol $r$ leads to $r$, every other symbol leads to $n$;
- from the state $r$, every symbol leads back to $r$.

**Solution.** Here are the rules for the Turing machine:

- start, $-$ $\rightarrow$ R, $s$
- $s$, $r$ $\rightarrow$ R, $r$
- $s$, $a$ $\rightarrow$ R, $n$
- $s$, $A$ $\rightarrow$ R, $n$
- $n$, $r$ $\rightarrow$ R, $r$
- $n$, $a$ $\rightarrow$ R, $n$
- $n$, $A$ $\rightarrow$ R, $n$
- $r$, $r$ $\rightarrow$ R, $r$
- $r$, $a$ $\rightarrow$ R, $r$
- $r$, $A$ $\rightarrow$ R, $r$
- $s$, $-$ $\rightarrow$ reject
- $n$, $-$ $\rightarrow$ accept
- $r$, $-$ $\rightarrow$ reject

**Tracing.**

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<tr>
<th></th>
<th>$-$</th>
<th>$A$</th>
<th>$a$</th>
<th>$a$</th>
<th>$-$</th>
<th>$\ldots$</th>
<th>start</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$-$</td>
<td>$A$</td>
<td>$a$</td>
<td>$a$</td>
<td>$-$</td>
<td>$\ldots$</td>
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<td>$a$</td>
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<td>$\ldots$</td>
<td>$n$</td>
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</tbody>
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\[
\begin{array}{ccc}
\vdash A & a & a & \ldots & n \\
\vdash A & a & a & \ldots & n \\
\vdash A & a & a & \ldots & \text{accept}
\end{array}
\]