

Solution to Homework Problem 20

Homework Problem 20. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes $n - 2$ for a binary number $n = 3$, show, step-by-step:

- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution. The rules of this Turing machine are as follows:

start, $- \rightarrow$ skip, R
 skip, 0 \rightarrow moving, R
 skip, 1 \rightarrow moving, R
 moving, 0 \rightarrow 1, R
 moving, 1 \rightarrow 0, L, back
 back, 0 \rightarrow L
 back, 1 \rightarrow L
 back, $- \rightarrow$ halt

1. At first, we have the following configuration:

| | | | | | | |
|---|---|---|---|---|---|-----|
| - | 1 | 1 | - | - | - | ... |
|---|---|---|---|---|---|-----|

start

Here, the left stack is empty, and the right stack has the form

| |
|---|
| - |
| 1 |
| 1 |

2. Then, the configuration changes to:

| | | | | | | |
|---|----------|---|---|---|---|-----|
| - | <u>1</u> | 1 | - | - | - | ... |
|---|----------|---|---|---|---|-----|

skip

Here, the two stacks have the following form:



To get to this configuration, we pop the symbol - (meaning black space) from the right stack and push it into the left stack.

3. Then, the configuration changes to:



Here, the left stack has the following form:



and the right stack has the following form:



To get to this configuration, we pop 1 from the right stack and push it into the left stack.

4. Then, the configuration changes to:



Here, the left stack has the following form:



and the right stack has the following form:



To get to this configuration, we replace 1 with 0, pop 0 from the right stack and push it into the left stack.

5. Then, the configuration changes to:



Here, the left stack is empty, and the right stack has the following form:



To get to this configuration, we pop blank from the left stack and push it into the right stack.

6. Then, the configuration changes to:



Here, we did not change anything on the tape, and we did not change the location of the head, so the stacks remain the same.