Solution to Homework Problem 20

Homework Problem 20. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes \( n - 2 \) for a binary number \( n = 3 \), show, step-by-step:

- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution. The rules of this Turing machine are as follows:

- start, \(-\) \(\rightarrow\) skip, R
- skip, 0 \(\rightarrow\) moving, R
- skip, 1 \(\rightarrow\) moving, R
- moving, 0 \(\rightarrow\) 1, R
- moving, 1 \(\rightarrow\) 0, L, back
- back, 0 \(\rightarrow\) L
- back, 1 \(\rightarrow\) L
- back, \(-\) \(\rightarrow\) halt

1. At first, we have the following configuration:

\[
\begin{array}{cccccccccc}
- & 1 & 1 & - & - & - & \ldots \\
\end{array}
\]

Here, the left stack is empty, and the right stack has the form

\[
\begin{array}{cccccccccc}
- & 1 & 1 & - & - & - & \ldots \\
\end{array}
\]

2. Then, the configuration changes to:

\[
\begin{array}{cccccccccc}
- & 1 & 1 & - & - & - & \ldots \\
\end{array}
\]

\[
\begin{array}{cccccccccc}
\end{array}
\]

Solution.
Here, the two stacks have the following form:

```
  
  1
  1
```

To get to this configuration, we pop the symbol – (meaning black space) from the right stack and push it into the left stack.

3. Then, the configuration changes to:

```
  1 1 1 - - - - - 
```

Here, the left stack has the following form:

```
  1
```

and the right stack has the following form:

```
  1
```

To get to this configuration, we pop 1 from the right stack and push it into the left stack.

4. Then, the configuration changes to:

```
  1 1 1 0 - - - - - 
```

Here, the left stack has the following form:

```
  1
```

and the right stack has the following form:

```
  1
  0
```

To get to this configuration, we replace 1 with 0, pop 0 from the right stack and push it into the left stack.

5. Then, the configuration changes to:

```
  1 1 1 0 - - - - - 
```

Here, the left stack is empty, and the right stack has the following form:

```
  1
  0
```

To get to this configuration, we pop blank from the left stack and push it into the right stack.

6. Then, the configuration changes to:

```
  1 1 1 0 - - - - - 
```

Here, we did not change anything on the tape, and we did not change the location of the head, so the stacks remain the same.