Automata Spring 2024, Test 1

Problem 1. Why do we need to study automata? Provide two main reasons.

Problem 2–4. Let us consider the automaton that has two states: c (cold) and h (hot); h is the starting state and it is also the final state. The symbols are s (sun) and w (wind). From each state, s leads to h and w lead to c.

Problem 2. Trace, step-by-step, how this finite automaton will check that the word sws belongs to this language. Use the tracing to find the parts x, y, and z of the word sws corresponding to the Pumping Lemma. Check that the "pumped" word $xyyz$ will also be accepted by this automaton.

Problem 3. Write down the tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ corresponding to this automaton:

- Q is the set of all the states,

- Σ is the alphabet, i.e., the set of all the symbols that this automaton can encounter;

- $\delta : Q \times \Sigma \to Q$ is the function that describes, for each state q and for each symbol s, the state $\delta(q, s)$ to which the automaton that was originally in the state q moves when it sees the symbol s (you do not need to describe all possible transitions this way, just describe two of them);

- q_0 is the staring state, and

- F is the set of all final states.

Problem 4. Use a general algorithm that we had in class to generate a context-free grammar corresponding to this automaton. Show how this grammar will generate the word sws.

Problem 5. Let A_1 be the automaton described in Problem 2. Let A_2 be an automaton that accepts only sequences of windy days. This automaton has two states: the starting state d which is also final, and the normal state n. The transitions are: as follows:

- from the start state, w lead back to the start state, while s leads to the state n;

- from the state n, any symbol leads back to this state.

Use the algorithm that we had in class to describe the following two new automata:

- the automaton that recognizes the union $A_1 \cup A_2$ of the two corresponding languages, and

- the automaton that recognizes the intersection of the languages A_1 and A_2.

Problem 6. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language $s^*ws^* \cup s^*$ – that corresponds to the sequences when there is at most one windy day:

- first, describe the automata for recognizing s and w;

- then, combine them into the automata for recognizing the Kleene star s^*;

- then, combine the automata for s^* and w into an automaton for recognizing concatenation s^*w;

- then, combine the automata for s^*w and s^* into an automaton for recognizing concatenation s^*ws^*;

- finally, combine the automata for recognizing s^*ws^* and s^* into the desired automaton that recognized their union.

Problem 7. Use the general algorithm to transform the resulting non-deterministic finite automaton into a deterministic one.

Problem 8–9. Use a general algorithm to transform the finite automaton from Problem 2 into the corresponding regular expression. Start with eliminating the state h.

Problem 10. In a class where all assignments are pass-fail, a student passes the class if he/she has more passes than fails. If we denote pass for an assignment by g (for "good") and fail by b (for "bad"), then the sequences gbg and bgg leads to pass, while a the sequence $ggbb$ leads to failing. Prove that the language L of all the sequence that lead to pass – i.e., that have more g's than b's – is not regular.