Automata, Computability, and Formal Languages
Spring 2024, Test 3

1-2. According to one of the health-promoting eating plans, 1/2 of the calories should come from carbs (C), 1/3 from fat (F), and 1/6 from proteins (P). For example, according to this plan, the sequence $PCFCFC$ is healthy, while the sequence $PPFC$ is not. Prove that the language H of all healthy sequences is not context-free.

3. The following Turing machine replaces each binary number with zeros:

- start, – → moving, R (here, – means blank)
- moving, 0 → R
- moving, 1 → 0, R
- moving, – → back, L
- back, 0 → L
- back, 1 → L
- back, – → halt

Trace it on the example of the binary number $110_2 = 6_{10}$. Explain how each step will be represented if we interpret the Turing machine as a finite automaton with two stacks.

4. Arithmetic operations on Turing machines:

a Design a Turing machine that adds 8 to a binary number.

b Trace your Turing machine, step-by-step, on the example of the number 3.

c Why in Turing machines (and in most actual computers) the representation of a binary number starts with the least significant digit?

5. The following finite automaton accepts healthy diets – i.e., diets that only contain fruits (f) and vegetables (v) and do not have cake (c). It has two states: the starting state h (healthy) which is also final, and the state u (unhealthy). Transitions are as follows:

- from the state h, symbols f and v lead back to h, while c leads to u;
- from the state u, each symbol leads back to u.

Use the general algorithm to transform this finite automaton into a Turing machine. Show, step-by-step, how your Turing machine will accept the string fv.

6. Give the formal definition of a feasible algorithm, and an explanation of what practically feasible means. Give two examples different from what we had in class:

- an example of a computation time which is formally feasible, but not practically feasible, and
- an example of a computation time which is practically feasible but not formally feasible.

These examples should be different from what we had in class, in homeworks, in last year's solutions. To make sure that your examples are different, use some numbers that have personal meaning to you – e.g., your birthday or digits from your car's licence plate.

7. What is P? What is NP? What does it means for a problem to be NP-hard? NP-complete? Give brief definitions. Give an example of an NP-complete problem: explain what is the input, what is the desired output. Is P equal to NP?