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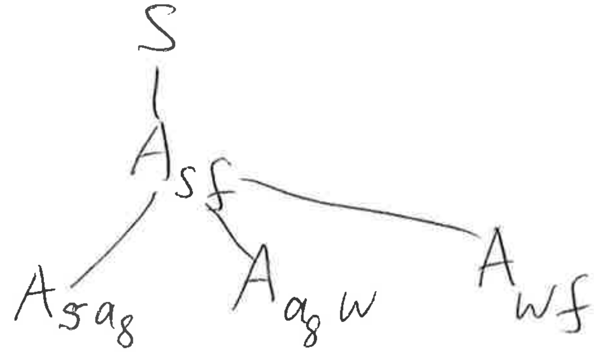
SOLUTION TO PROBLEM 11 FALL 2024

Acceptance of the word RSR has the following form (see solution to Problem 6):

read			R		S			R	
state	s	w	a_1	w	a_7	a_8	w	w	f
stack		\$	\$	R \$	\$		N		

There is only one final state f in this PDA, so we start with rule

In the tracing of acceptance, there are two intermediate states with empty stack, so we use transitivity rule:



(2) Let us deal with the three transitions A_{sa_8}, $A_{a_8 w}$, and $A_{w,f}$ one by one.

A_{sa_8}. This transition has the following form:

		R		S'	
s	w	a_1	w	a_7	a_8
	\$	\$	R \$	\$	

The first symbol we push is \$, so let us write down the rules used to push \$ and then to pop \$:

$$s \xrightarrow{\varepsilon,\varepsilon \to \$} w \qquad a_7 \xrightarrow{S,\$ \to \varepsilon} a_8$$

By using the general pattern, we get the rule $A_{sa_8} \longrightarrow \varepsilon A_{wa_7} \varepsilon$, i.e., the rule $A_{sa_8} \longrightarrow A_{wa_7}$. Let us write it down:

$$\begin{array}{c} A_{sa_8} \\ | \\ A_{wa_7} \end{array}$$

③ We took care of \$ and of the two transitions:

		R		S	
~~S~~	w	a_1	w	a_7	~~a_8~~
	~~\$~~	~~\$~~	R	~~\$~~	
			~~\$~~		

Now, we need to describe the transition from w to a_7.
Here, we again have an intermediate state (a_1) with an empty stack, so we again need to use transitivity rule:

$$A_{wa_7} \longrightarrow A_{wa_1} A_{a_1 a_7}$$

Now the tree gets the following form:

$$\begin{tikzcd}[column sep=small] & A_{sa_8} \arrow[d, dash] & \\ & A_{wa_7} \arrow[dl, dash] \arrow[dr, dash] & \\ A_{wa_1} & & A_{a_1 a_7} \end{tikzcd}$$

Let us consider the transitions A_{wa_1} and $A_{a_1 a_7}$ one by one

④ $\underline{A_{wa_1}}$

	R
w	a_1
$\cancel{\$}$	$\cancel{\$}$

For going from w to a_1, we use the rule $R, \varepsilon \to \varepsilon$ (actually, in Problem 6, we use the rule $R, \$ \to \$$ but in reality nothing changes in the stack, so it is the same as using the rule $R, \varepsilon \to \varepsilon$);

$$(w) \xrightarrow{R, \varepsilon \to \varepsilon} (a_1)$$

We have only one rule, so we add a fictitious rule

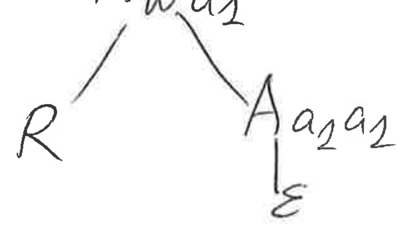
$$(w) \xrightarrow{R, \varepsilon \to \varepsilon} (a_1) \qquad (a_1) \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} (a_1)$$

Now we can form a rule

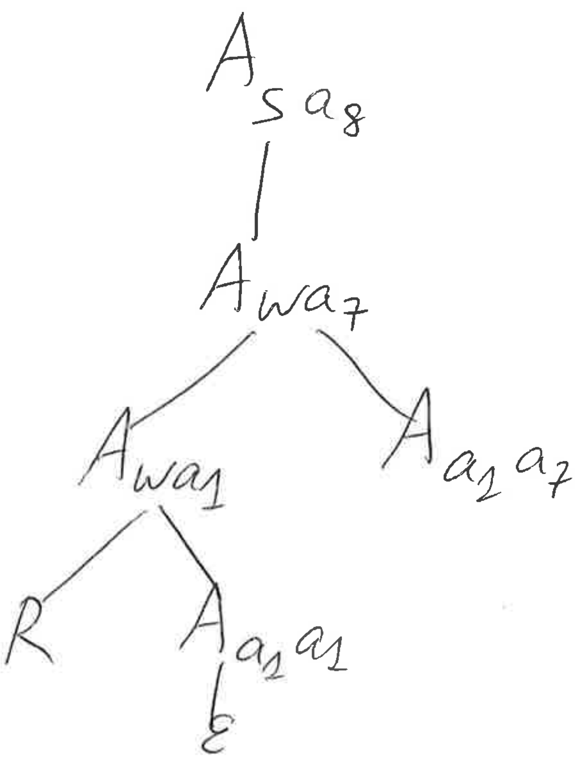
$A_{wa_1} \to R A_{a_1 a_1} \varepsilon$, i.e. $A_{wa_1} \to R A_{a_1 a_1}$

	R
$w \to$	a_1

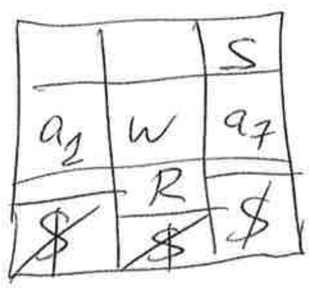
what remains is a transition from a_1 with empty stack to a_1 with empty stack, so we use the rule $A_{a_1 a_1} \to \varepsilon$. Now the transition from A_{wa_1} takes the form



(5) and the transition A_{Sa_8} takes the form

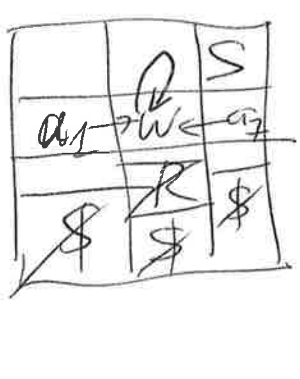


$\underline{A_{a_1a_7}}$:



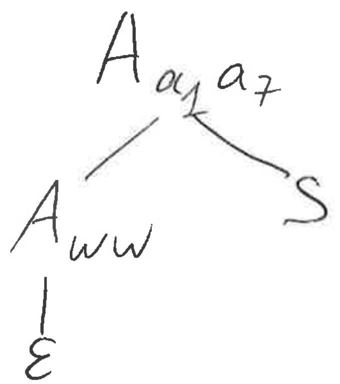
we push R, then we pop R

$A_{a_1a_7} \to \varepsilon A_{ww} S$, i.e. $A_{a_1a_7} \to A_{ww} S$

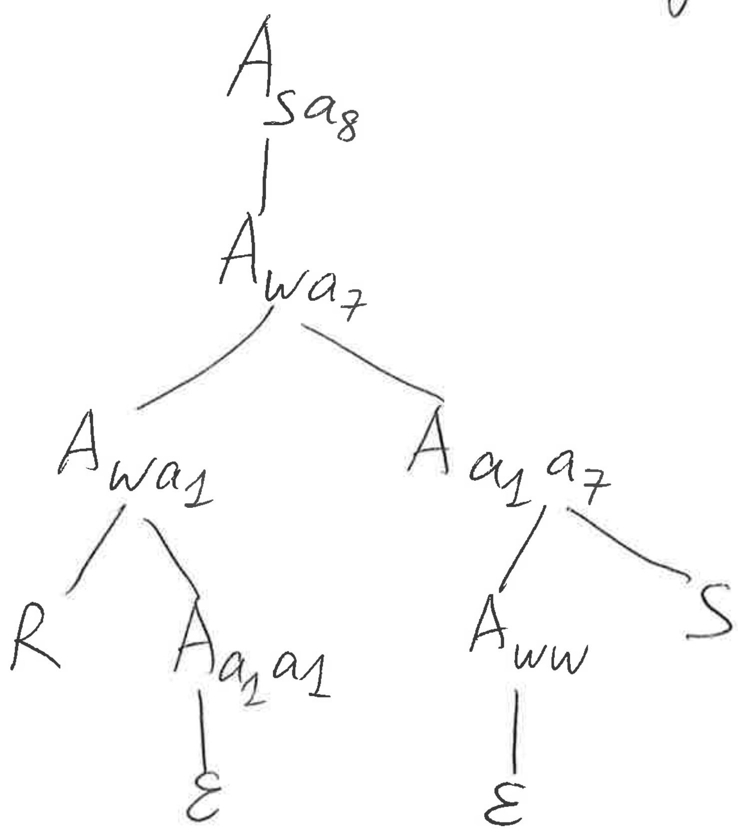


We took care of R and of transitions, so we add $A_{ww} \to \varepsilon$

6 So, for $A_{a_1 a_7}$, we have:



and the transition from $A_{s a_8}$ takes the following form:



⑦ $\underline{A_{a_8 w}}$:

		R
a_8	w	w
	N	

we push N, then pop it.

The rules are:

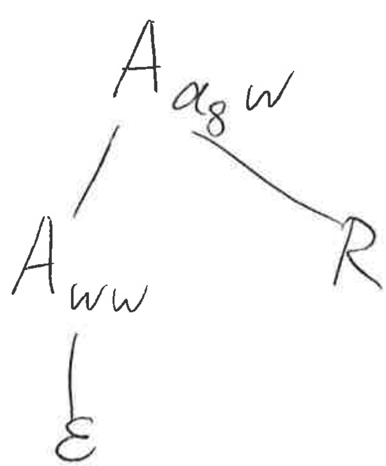
$a_8 \xrightarrow{\varepsilon,\varepsilon \to N} w$ $\qquad$ $w \xrightarrow{R, N \to \varepsilon} w$

So we get $A_{a_8 w} \to \varepsilon A_{ww} R$, i.e.

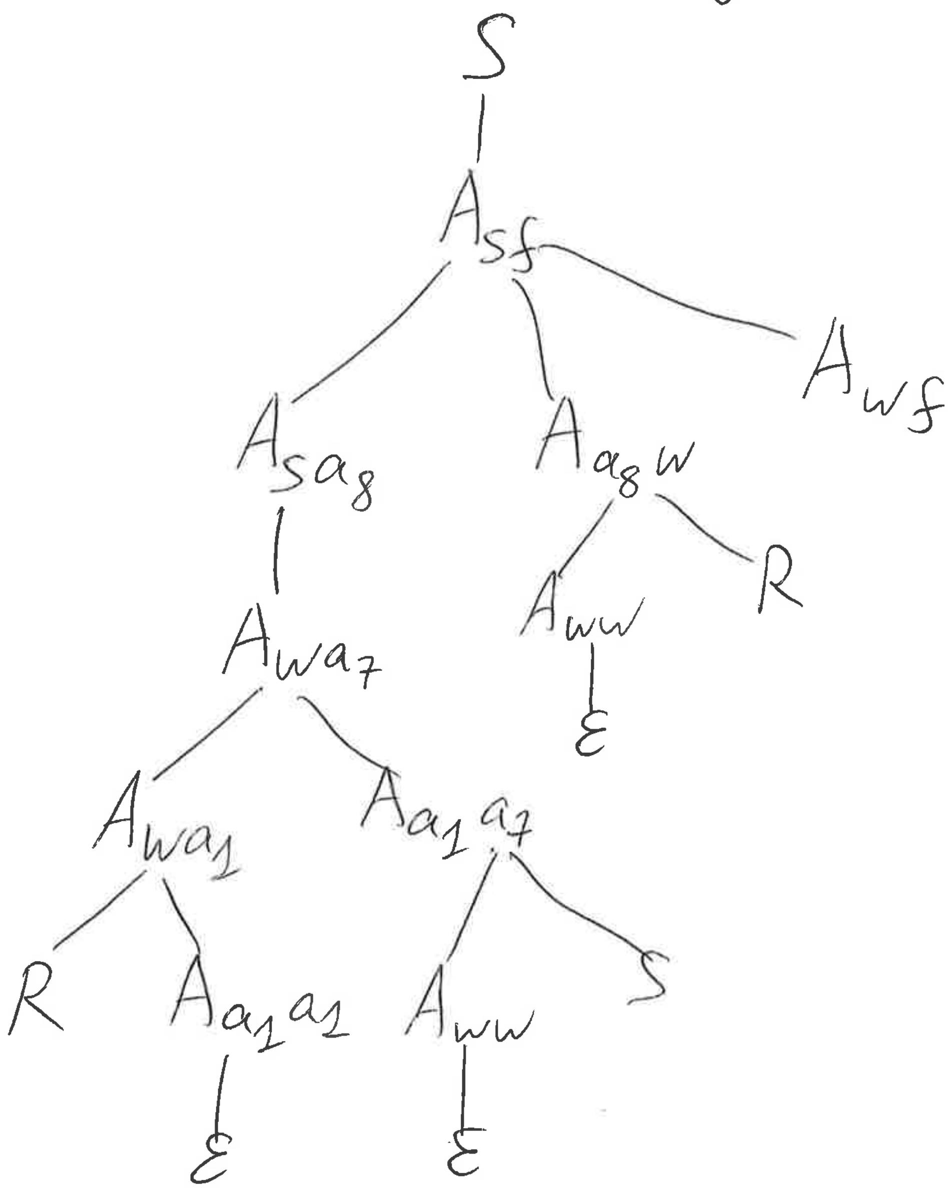
$A_{a_8 w} \to A_{ww} R$.

So we took care of N and of both transitions, so now we go from w with empty stack to itself; the rule is $A_{ww} \to \varepsilon$.

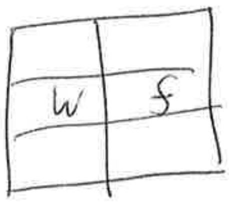
So, the subtree coming from $A_{a_8 w}$ has the form



(8) The general derivation so far has the following form:



(9) Finally, let us deal with the transition $\underline{A_{wf}}$



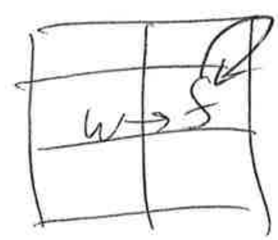
We have a single rule

$$w \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} f$$

So we need to supplement it with a fictitious rule

$$w \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} f \qquad f \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} f$$

that leads to $A_{wf} \to \varepsilon A_{ff} \varepsilon$, i.e. $A_{wf} \to A_{ff}$. The remaining transition is from f to f

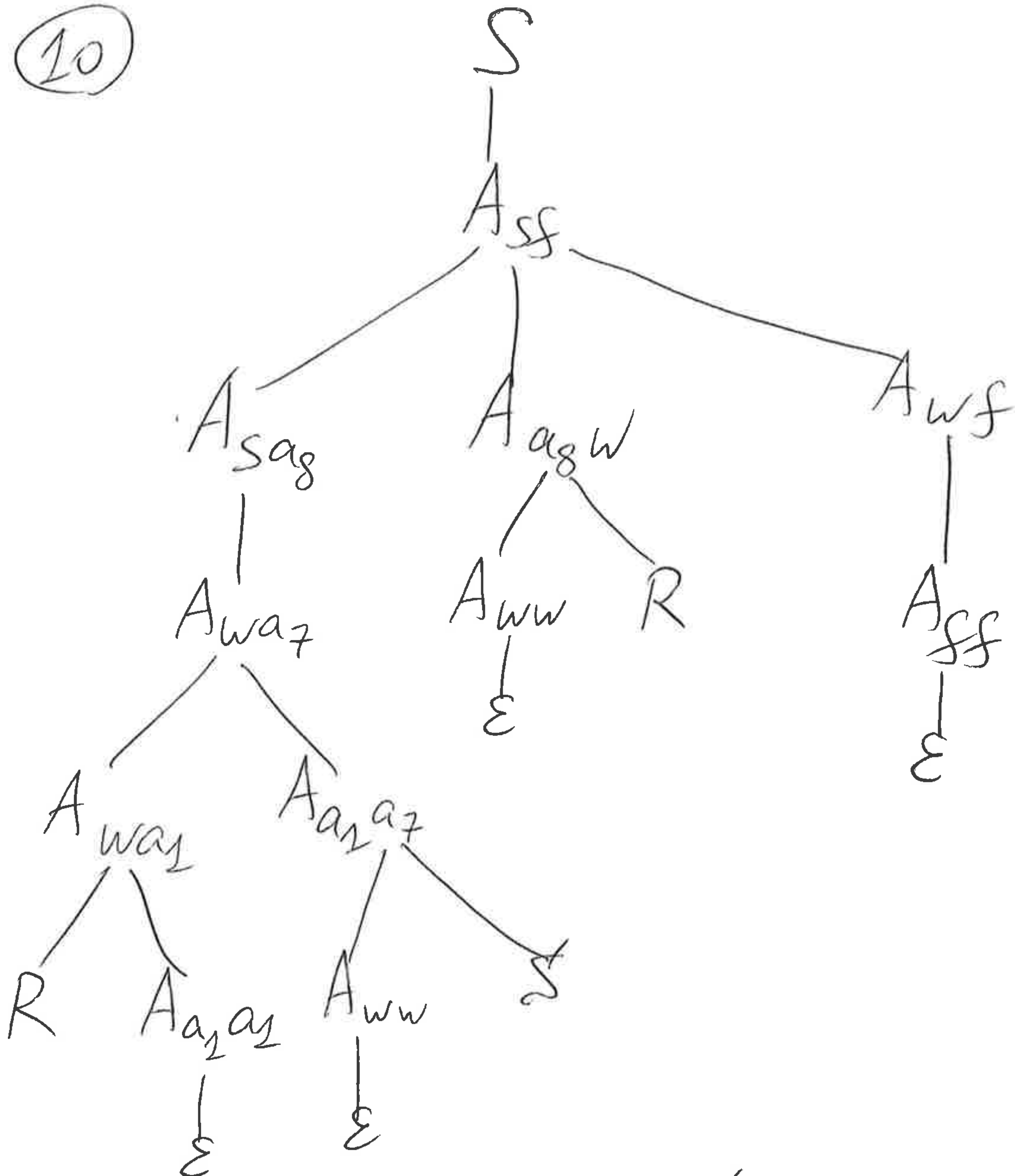


so we use the rule $A_{ff} \to \varepsilon$

So, the tree coming from A_{wf} has the form

$$\begin{array}{c} A_{wf} \\ | \\ A_{ff} \\ | \\ \varepsilon \end{array}$$

and the resulting generation has the form



We have indeed generated the word RSR in the corresponding grammar.