Solution to Homework Problem 15

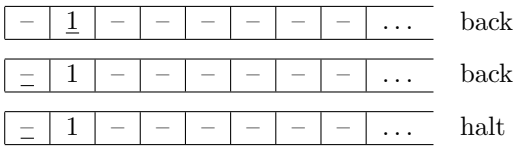
Task. Design a Turing machine that, given a unary number n which is great than or equal to 4, subtracts 4 from this number. Test it, step-by-step, on the example of $n = 5$.

Idea. This is similar to subtracting 1 to a unary number. The only difference is that after we subtract 1, we need to subtract three more 1s. Then, we go back.

Solution. Here are the rules for the Turing machine:
start, – → R, moving
moving, 1 → R
moving, – → L, delete1st, L
delete1st, 1 → –, delete2nd, L
delete2nd, 1 → –, delete3rd, L
delete3rd, 1 → –, delete4th, L
delete4th, 1 → –, back, L
back, 1 → L
back, – → halt

Tracing.

$\underline{-}$	1	1	1	1	1	–	–	...	start
–	$\underline{1}$	1	1	1	1	–	–	...	moving
–	1	$\underline{1}$	1	1	1	–	–	...	moving
–	1	1	$\underline{1}$	1	1	–	–	...	moving
–	1	1	1	$\underline{1}$	1	–	–	...	moving
–	1	1	1	1	$\underline{1}$	–	–	...	moving
–	1	1	1	1	1	$\underline{-}$	–	...	moving
–	1	1	1	1	$\underline{1}$	–	–	...	delete1st
–	1	1	1	$\underline{1}$	–	–	–	...	delete2nd
–	1	1	$\underline{1}$	–	–	–	–	...	delete3rd
–	1	$\underline{1}$	–	–	–	–	–	...	delete4th


– 1 – – – – – – ... back
– 1 – – – – – – ... back
– 1 – – – – – – ... halt