

Solution to Homework Problem 16

Task. Design a Turing machine that, given a binary number n , adds 16 to this number. Test it, step-by-step, on the example of $n = 0$.

Solution. Here are the rules for the Turing machine:

start, $- \rightarrow R$, skip1st
 skip1st, $0 \rightarrow R$, skip2nd
 skip1st, $1 \rightarrow R$, skip2nd
 skip1st, $- \rightarrow 0, R$, skip2nd
 skip2nd, $0 \rightarrow R$, skip3rd
 skip2nd, $1 \rightarrow R$, skip3rd
 skip2nd, $- \rightarrow 0, R$, skip3rd
 skip3rd, $0 \rightarrow R$, skip4th
 skip3rd, $1 \rightarrow R$, skip4th
 skip3rd, $- \rightarrow 0, R$, skip4th
 skip4th, $0 \rightarrow R$, moving
 skip4th, $1 \rightarrow R$, moving
 skip4th, $- \rightarrow 0, R$, moving
 moving, $1 \rightarrow 0, R$, moving
 moving, $0 \rightarrow 1, L$, back
 moving, $- \rightarrow 1, L$, back
 back, $1 \rightarrow L$
 back, $0 \rightarrow L$
 back, $- \rightarrow \text{halt}$

Tracing. We start with the number $0_{10} = 0_2$ which is represented as 0.

_	0	-	-	-	-	-	...	start
-	<u>0</u>	-	-	-	-	-	...	skip1st
-	1	_	-	-	-	-	...	skip2nd
-	0	0	_	-	-	-	...	skip3rd
-	0	0	0	_	-	-	...	skip4th
-	0	0	0	0	_	-	...	moving
-	0	0	0	<u>0</u>	1	-	...	back
-	0	0	<u>0</u>	0	1	-	...	back

-	0	<u>0</u>	0	0	1	-	...	back
-	<u>0</u>	0	0	0	1	-	...	back
<u>-</u>	0	0	0	0	1	-	...	back
<u>-</u>	0	0	0	0	1	-	...	halt