

Solution to Problem 6

Task: Show, step by step, how the following pushdown automaton – that checks whether a word consisting of letters S and R corresponds to a situation in which there are at least twice as many rest hours as study hours – will accept the word RSR . This pushdown automaton has three main states:

- the starting state s ,
- the working state w , and
- the final state f ,

and several intermediate states.

In the stack, in addition to the bottom symbol $\$$, we have:

- either one or several N s – indicating the difference between twice the number of study hours and the number of rest hours so far (if this difference is positive),
- or one or several R s – indicating the difference between the number of rest hours and twice the number of study hours (if this difference is positive).

The transitions are as follows:

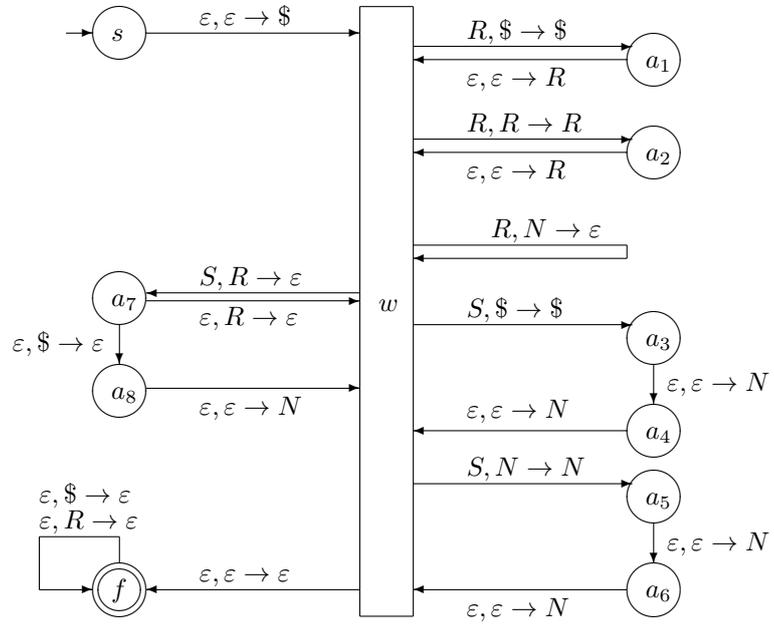
- From s to w , the transition is $\varepsilon, \varepsilon \rightarrow \$$.
- From w to f , the transition is: $\varepsilon, \varepsilon \rightarrow \varepsilon$.
- From f to f , we have two transitions: $\varepsilon, R \rightarrow \varepsilon$ and $\varepsilon, \$ \rightarrow \varepsilon$.

From w to w , we have the following transitions:

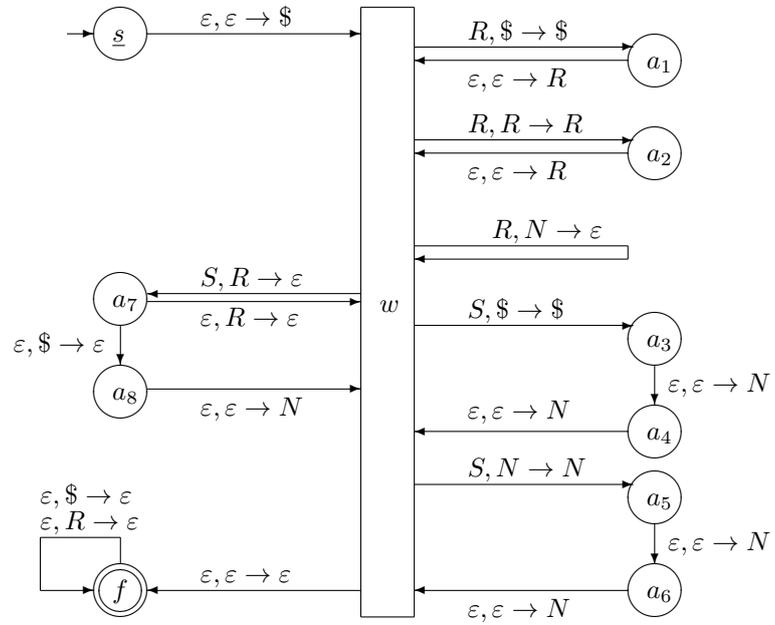
- If we see the symbol R and $\$$ is on top of the stack, we keep the dollar sign and add R to the stack, i.e., we have transition $R, \$ \rightarrow \$$ that brings us to an intermediate state a_1 , and then the transition $\varepsilon, \varepsilon \rightarrow R$ that brings us back to the working state.
- If we see the symbol R and R is on top of the stack, we keep the top R and add another R to the stack, i.e., we have transition $R, R \rightarrow R$ that brings us to an intermediate state a_2 , and then the transition $\varepsilon, \varepsilon \rightarrow R$ that brings us back to the working state.

- If we see the symbol R and N is on top of the stack, we delete the top N , i.e., we have transition $R, N \rightarrow \varepsilon$.
- If we see the symbol S and $\$$ is on top of the stack, we keep the dollar sign and add two N s to the stack, i.e., we have transition $S, \$ \rightarrow \$$ that brings us to an intermediate state a_3 , then the transition $\varepsilon, \varepsilon \rightarrow N$ that brings us to an intermediate state a_4 , and then the transition $\varepsilon, \varepsilon \rightarrow N$ that brings us back to the working state.
- If we see the symbol S and N is on top of the stack, we keep the top N and add two more N s to the stack, i.e., we have transition $S, N \rightarrow N$ that brings us to an intermediate state a_5 , then the transition $\varepsilon, \varepsilon \rightarrow N$ that brings us to an intermediate state a_6 , and finally the transition $\varepsilon, \varepsilon \rightarrow N$ that brings us back to the working state.
- If we see the symbol S and R is on top of the stack, we first delete the top R from the stack, i.e., we have transition $S, R \rightarrow \varepsilon$, and move to an intermediate state a_7 .
 - If in this state, we see R on top of stack, we delete this R and go back to the working state; the transition is $\varepsilon, R \rightarrow \varepsilon$.
 - if in this state, we see $\$$ on top of the stack, we add N , i.e., first we apply the transition $\varepsilon, \$ \rightarrow \varepsilon$ and go to the intermediate state a_8 , and then we apply the transition $\varepsilon, \varepsilon \rightarrow N$, and go to the working state.

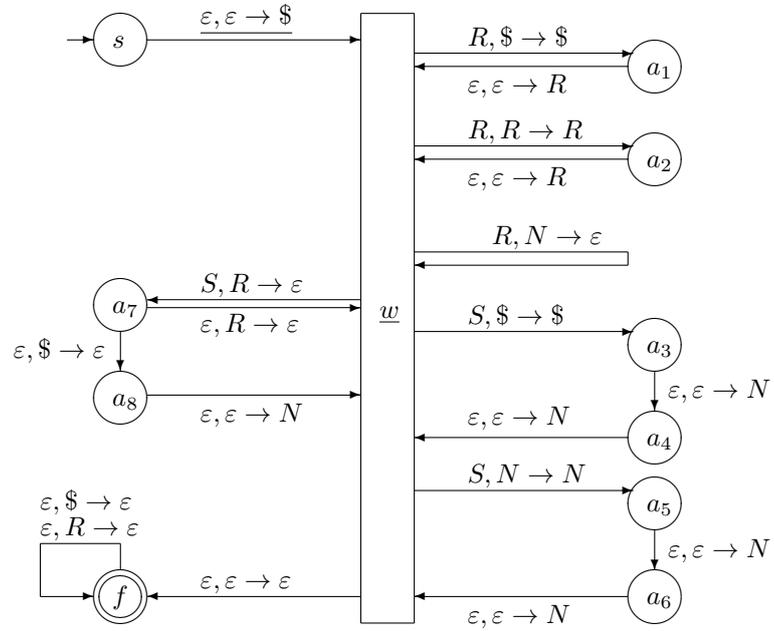
Solution. Our pushdown automaton has the following form:



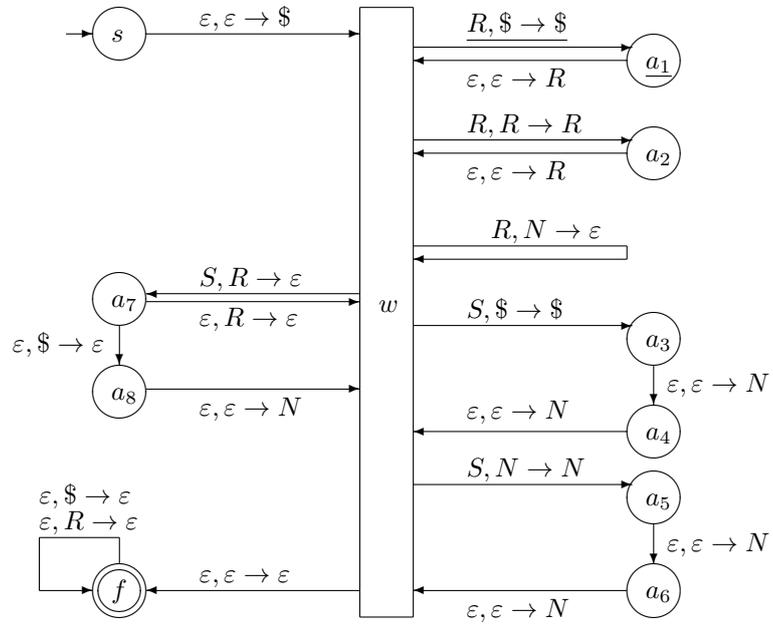
We start in the starting state s with an empty stack:

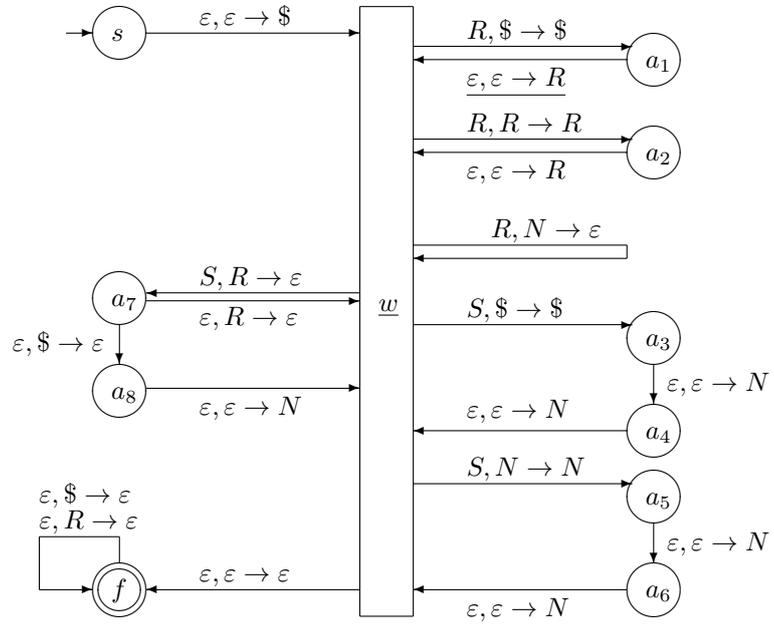


First, we add dollar sign to the stack and go to the working state:

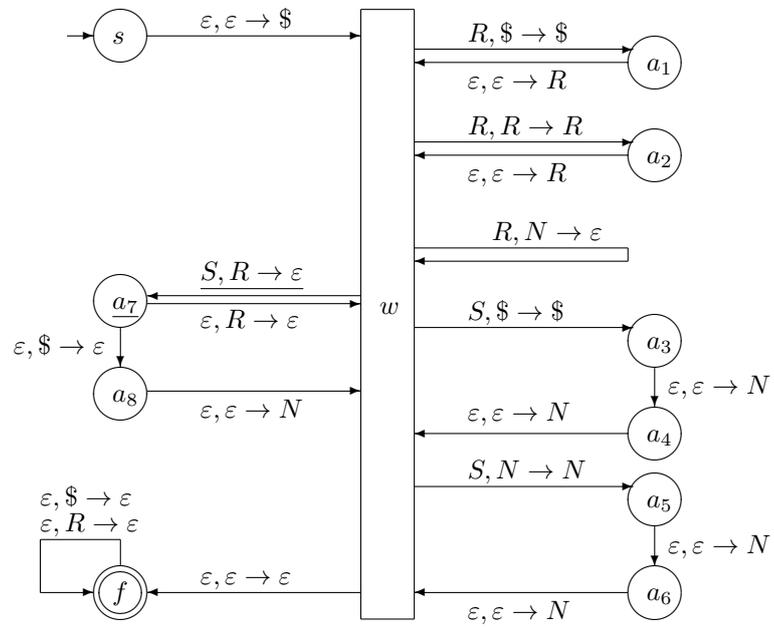


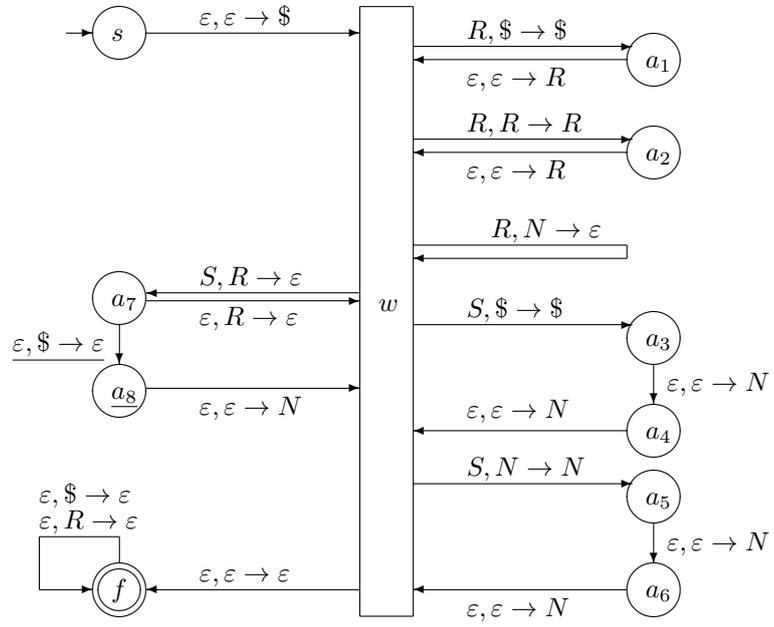
Then, we see the symbol R and first go to a_1 , then back to the working state:

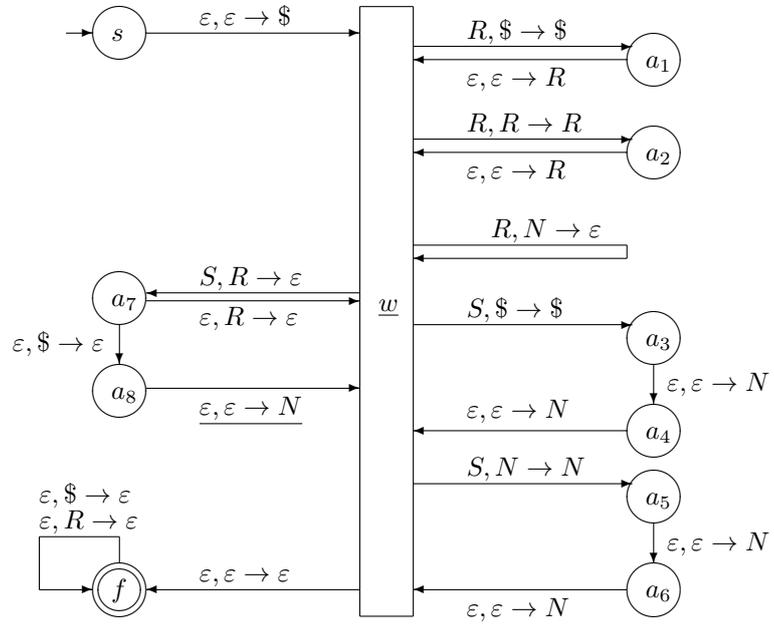




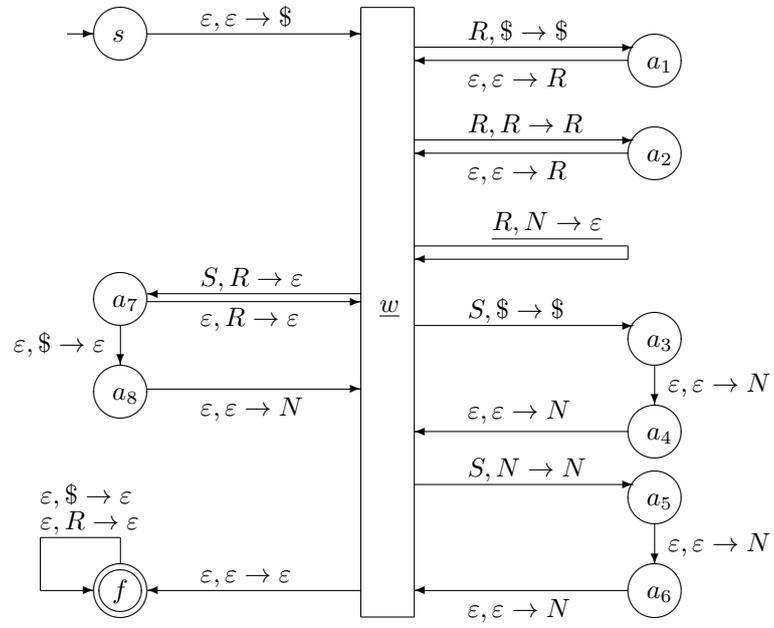
Then, we see symbol S , go to a_7 , then to a_8 , and then to the working state:



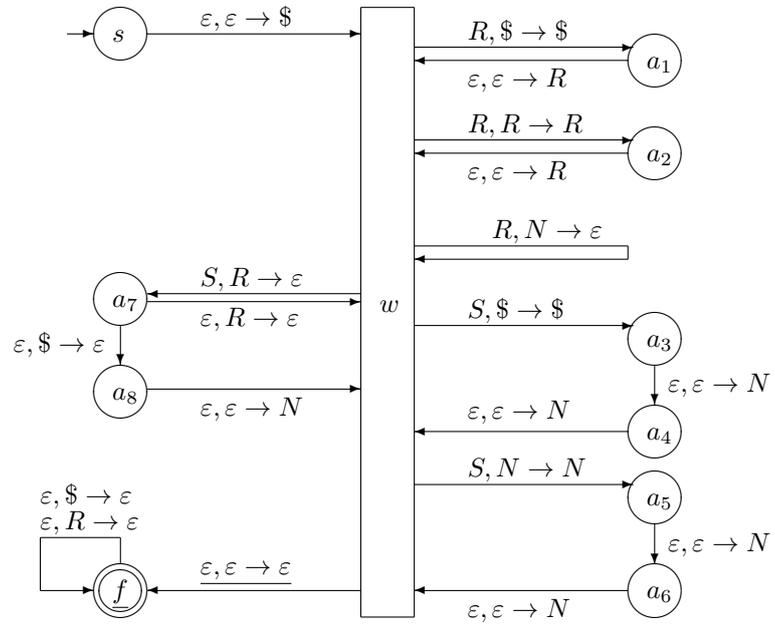




Now, we see R , so we delete N from the stack:



Now, we can jump to the final state:



Now, we are in the final state with an empty stack, so the word RSR is accepted.

To illustrate these transitions, let us list all the symbols we read, all the states that this automaton goes through, and under each state, the contents of the corresponding stack:

read			R		S			R	
state	s	w	a_1	w	a_7	a_8	w	w	f
stack		$\$$	$\$$	R	$\$$		N		