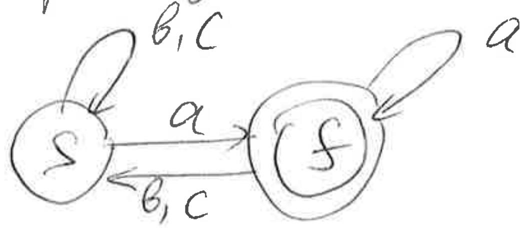
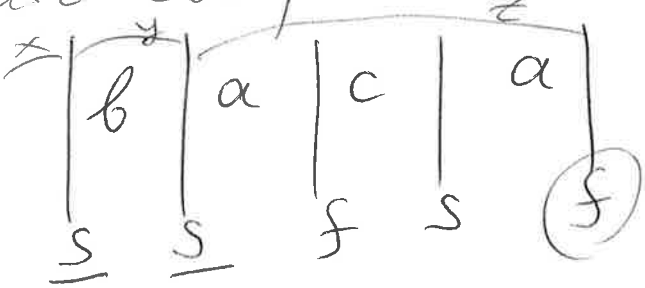
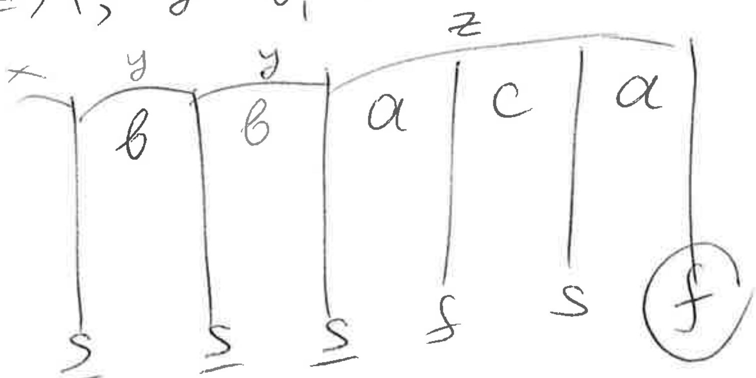
AUTOMATA TEST 1
SOLUTIONS
SPRING 2025

(1.) * To help develop a general understanding of which general problems are solvable and which are not
* To understand how programs are compilled



(2) 

$x = \Lambda$, $y = b$, $z = aca$

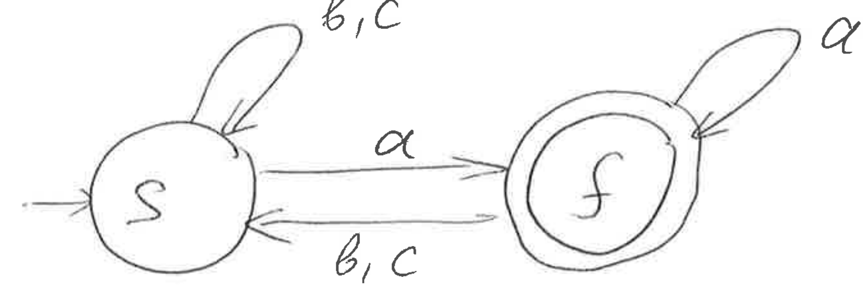


(3) $Q = \{s, f\}$, $\Sigma = \{a, b, c\}$, $q_0 = s$, $F = \{f\}$

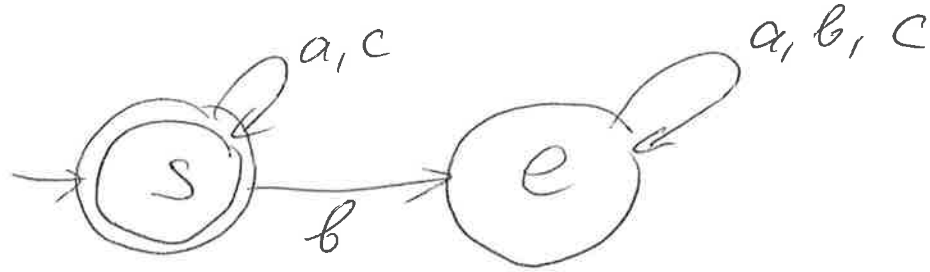
	s	f
a	f	f
b,c	s	s

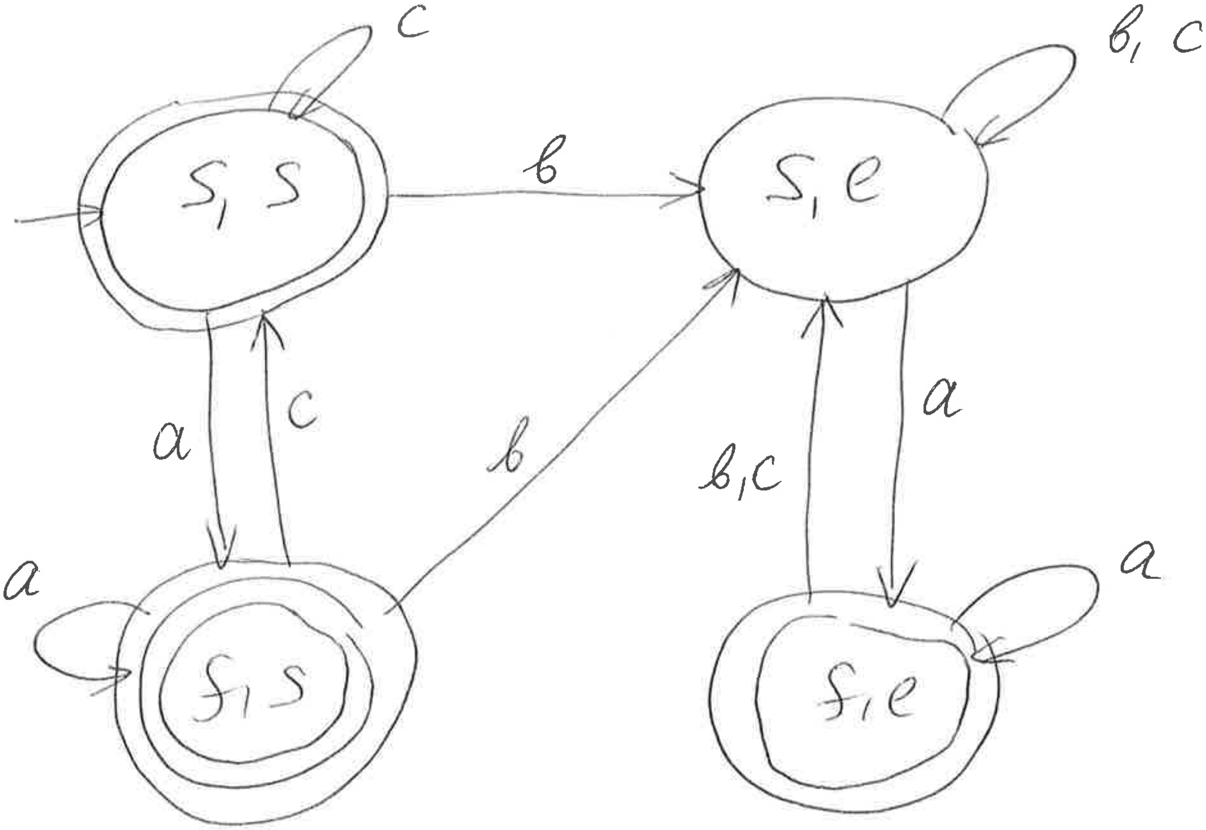
4

A_1:



A_2:



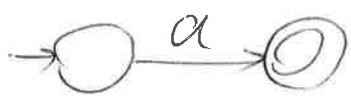


5 6

p.1

$(a \vee b \vee c)^* a$

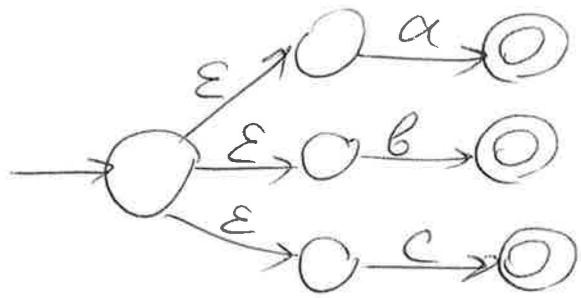
a



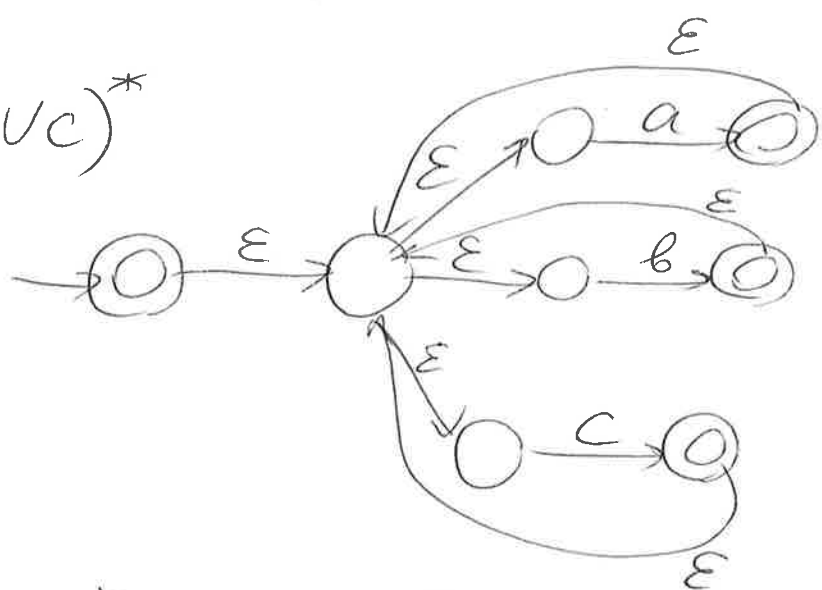
b

c

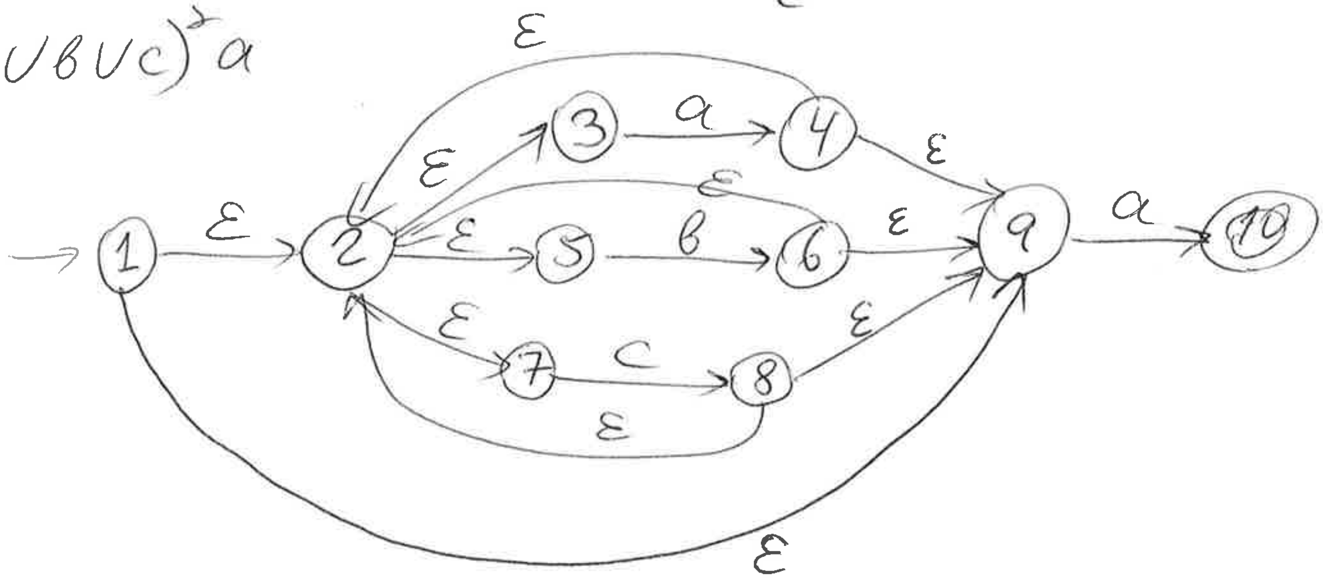
$a \vee b \vee c$



$(a \vee b \vee c)^*$

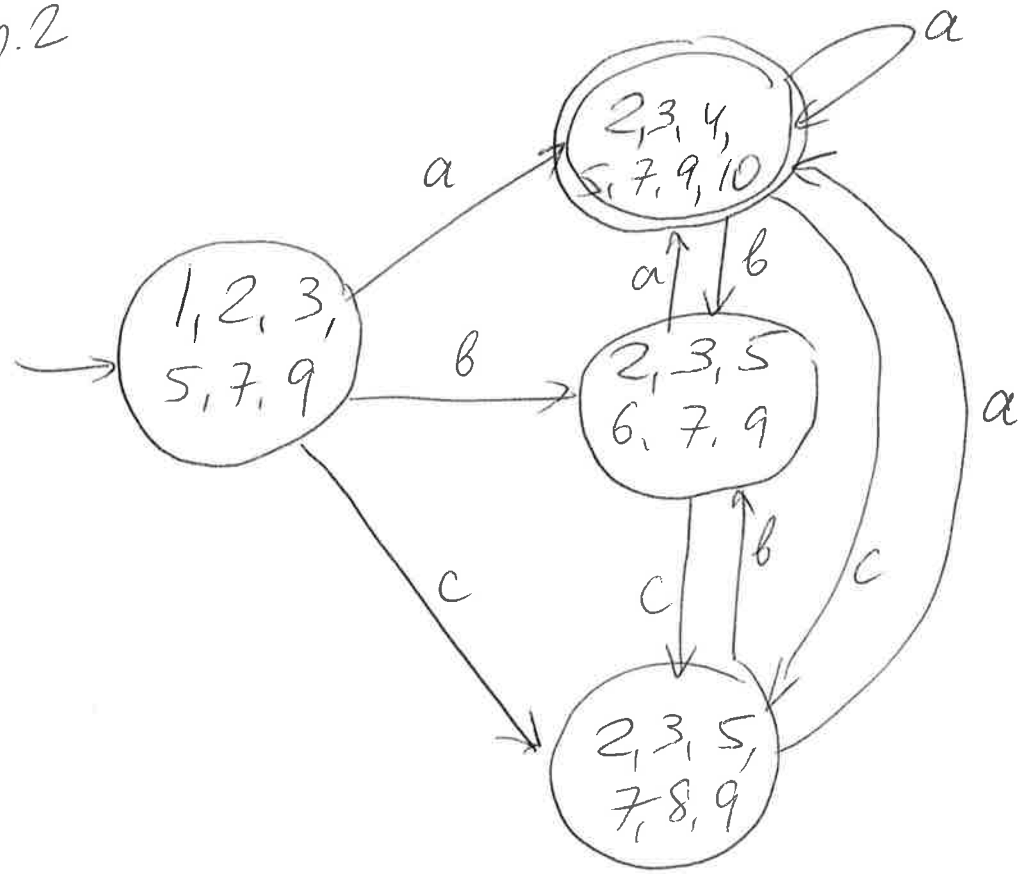


$(a \vee b \vee c)^* a$

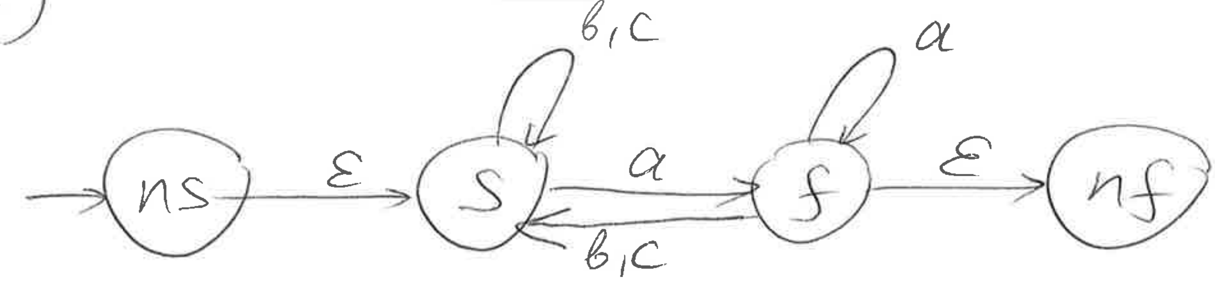


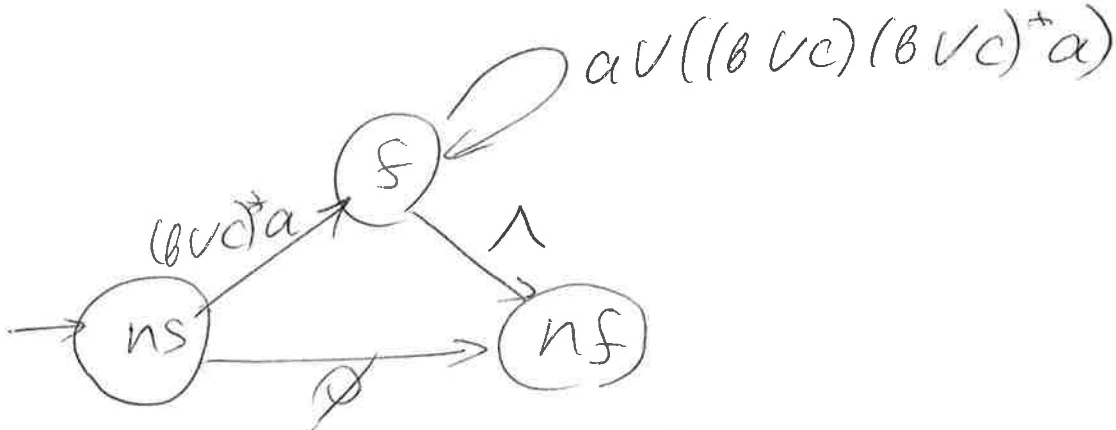
6.

p.2



7-8





$$R'_{ns,f} = R_{ns,f} \cup (R_{ns,s} R_{s,s}^{*} R_{s,f}) =$$
$$\emptyset \cup (\Lambda (b \cup c)^{*} a) = (b \cup c)^{*} a$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,s} R_{s,s}^{*} R_{s,nf}) =$$
$$\emptyset \cup (\Lambda (b \cup c)^{*} \emptyset) = \emptyset \cup \emptyset = \emptyset$$

$$R'_{f,nf} = R_{f,nf} \cup (R_{\cancel{ns}f,s} R_{s,s}^{*} R_{s,nf}) =$$
$$\Lambda \cup ((b \cup c)(b \cup c)^{*} \emptyset) = \Lambda \cup \emptyset = \Lambda$$

$$R'_{f,f} = R_{f,f} \cup (R_{f,s} R_{s,s}^{*} R_{s,f}) =$$
$$a \cup ((b \cup c)(b \cup c)^{*} a)$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,f} R_{f,f}^{*} R_{f,nf}) =$$
$$\emptyset \cup ((b \cup c)^{*} a (a \cup ((b \cup c)(b \cup c)^{*} a))^{*} \Lambda =$$
$$\underline{(b \cup c)^{*} a (a \cup ((b \cup c)(b \cup c)^{*} a))^{*}}$$

(9, 10) Proof by contradiction. Let us assume that L is regular. Then by pumping lemma

$$\exists p \, \forall w_{\in L} (len(w) \geq p \rightarrow \exists x, y, z : (w = xyz \, \& \, len(y) > 0 \, \& \, len(xy) \leq p \, \& \, \forall i (xy^i z \in L)))$$

Let us take the word

$$w = a^p b^p c^p = \underbrace{a \ldots a}_{p \text{ times}} \underbrace{b \ldots b}_{p \text{ times}} \underbrace{c \ldots c}_{p \text{ times}}$$

The length of this word is $p + p + p \geq p$, so by pumping lemma, this word can be represented as xyz with $len(xy) \leq p$. The word starts with xy, and the length of xy is $\leq p$, so xy is among first p symbols of w, and these symbols are all a's. So, y has only a's.

Thus, when we go from xyz to $xyyz$, we add a's but we do not add any b's or c's. We started with xyz that has equal # of a's, b's, and c's. Thus, in $xyyz$ we have more a's than b's or c's – the balance is disrupted, so $\underline{xyyz \notin L}$.

On the other hand, by pumping lemma for $i = 2$, $\underline{xyyz \in L}$. The contradiction shows that our assumption is false, and L is not regular.