Test 1, Automata, Fall 2025

Problem 1. Why do we need to study automata? Provide two main reasons.

Problems 2–4. Let us consider a 2-state automaton that accepts only words that end with a period. This automaton has two states: start state s and final state f, and three symbols: a, b, and the period. From both states, letters a and b lead to s, and the period . lead to f.

Problem 2. Trace, step-by-step, how this finite automaton will check that the word *abba.* belongs to this language. Use the tracing to find the parts x, y, and z of the word *abba.* corresponding to the Pumping Lemma. Check that the "pumped" word $xyyz$ will also be accepted by this automaton.

Problem 3. Write down the tuple $\langle Q, \Sigma, \delta, q_0, F\rangle$ corresponding to this automaton:

- Q is the set of all the states,
- Σ is the alphabet, i.e., the set of all the symbols that this automaton can encounter;
- $\delta : Q \times \Sigma \to Q$ is the function that describes, for each state q and for each symbol s, the state $\delta(q, s)$ to which the automaton that was originally in the state q moves when it sees the symbol s (you do not need to describe all possible transitions this way, just describe two of them);
- q_0 is the staring state, and
- F is the set of all final states.

Problem 4. Let A_1 be the automaton described in Problem 2. Let A_2 be an automaton that accepts only sequences that *do not* end with the period. This automaton has the same states and the same transitions as A_1, the only difference is that now s is the final state, and f is not a final state. Use the algorithm that we had in class to describe the following two new automata:

- the deterministic automaton that recognizes the union $A_1 \cup A_2$ of the two corresponding languages, and
- the deterministic automaton that recognizes the intersection of the languages A_1 and A_2.

Problem 5. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language $(a \cup b \cup .)^*(a \cup b)$ (that corresponds to all the words that do not end with a period):

- first, describe automata for recognizing a, b, and .;
- then, use the automata for a, b, and . to design an automaton for recognizing $a \cup b$ and an automaton for recognizing $a \cup b \cup .$;
- then, transform the automaton for $a \cup b \cup -$ into an automaton for recognizing the Kleene $(a \cup b \cup .)^*$;
- then, use the automata for $(a \cup b \cup .)^*$ and $a \cup b$ to design an automaton for concatenation $(a \cup b \cup .)^*(a \cup b)$.

Problem 6. Use the general algorithm to transform the resulting non-deterministic finite automaton into a deterministic one.

Problem 7-8. Use a general algorithm to transform the finite automaton from Problem 2 into the corresponding regular expression. Start with eliminating the state s.

Problem 9-10. Prove that the language L of all the words that have equal number of a's and b's and have exactly one period at the end – is not regular.