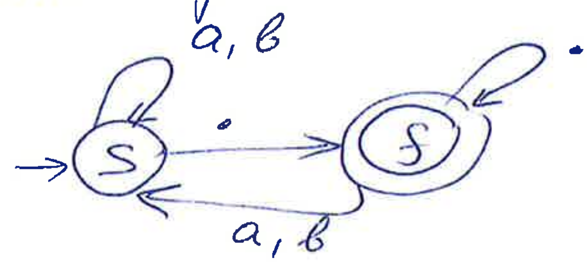
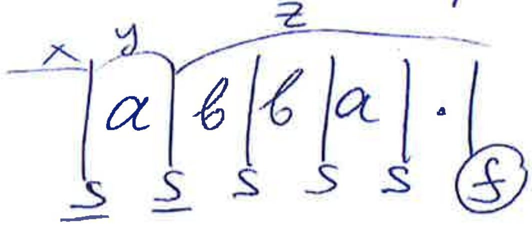
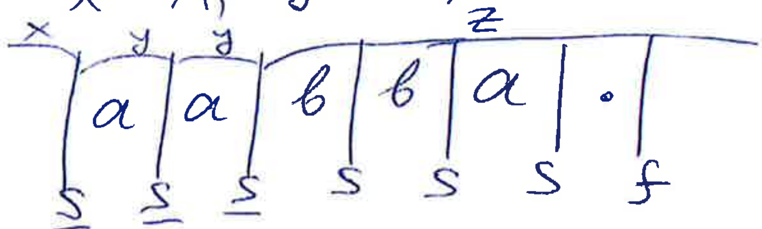
# AUTOMATA TEST I
# SOLUTIONS
# FALL 2025

① * To help develop a general understanding of which problems are solvable and which are not
* To understand how problems are compiled




$x = \Lambda$, $y = a$, $z = bba$.




③ $Q = \{s, f\}$  $\Sigma = \{a, b, .\}$, $q_0 = s$, $F = \{f\}$

| | s | f |
|---|---|---|
| a, b | s | s |
| . | f | f |

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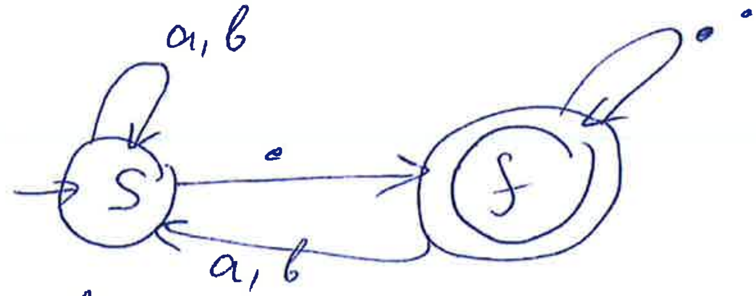
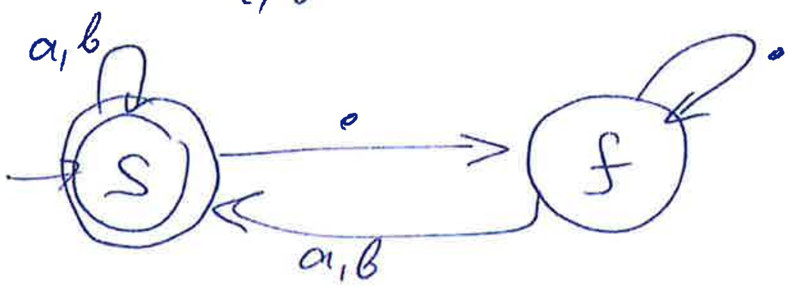
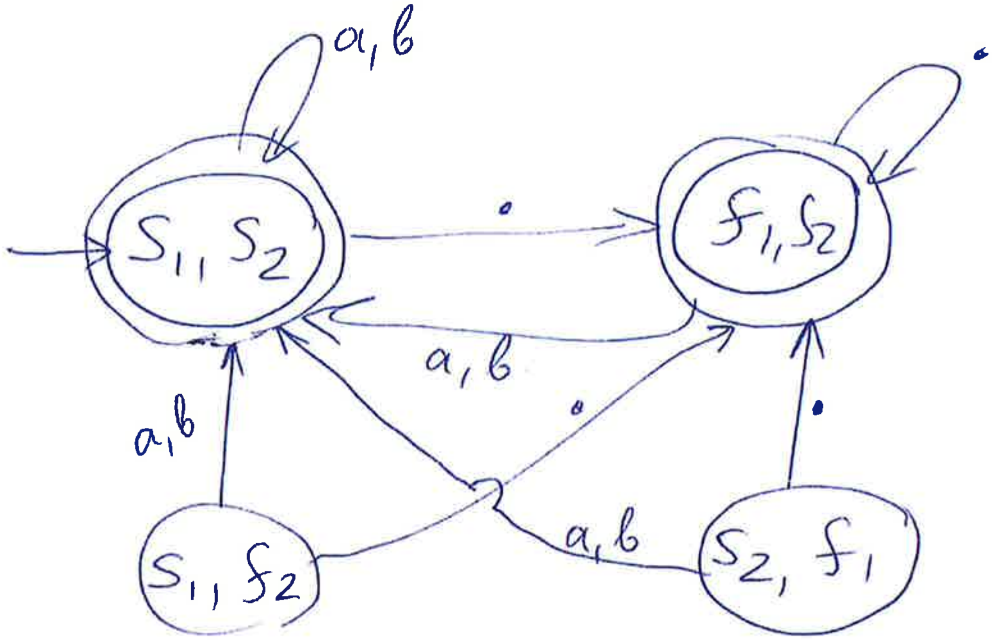
$A_1$




$A_2$

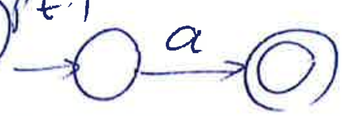



(It is OK not to draw the last two states since we cannot reach them from the starting state)

(5) (6) St.1

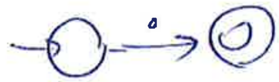
$a$



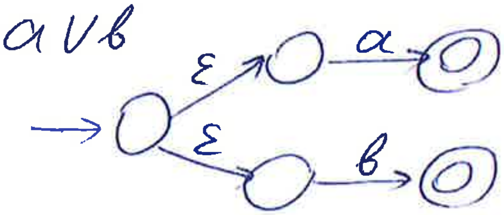


$b$

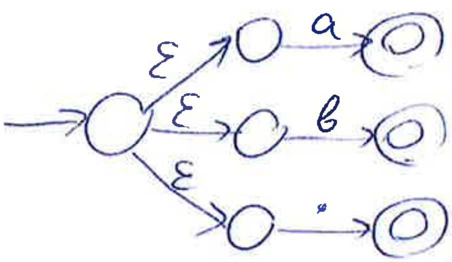
$\circ$



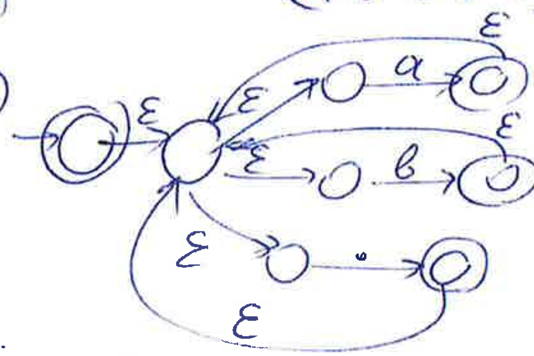
$a \vee b$



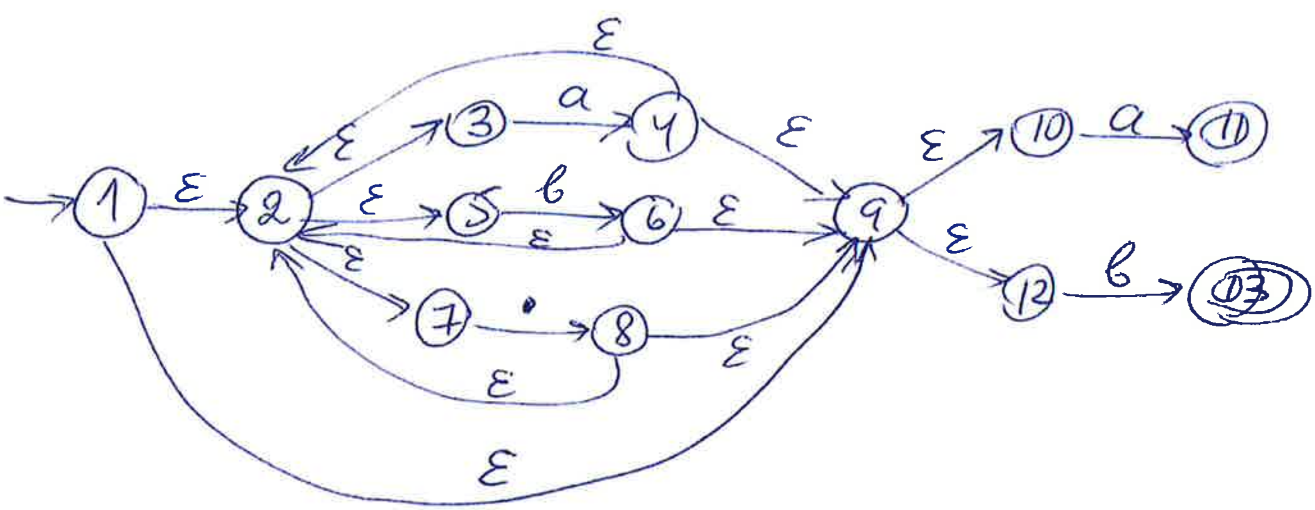

$a \vee b \vee \circ$




$(a \vee b \vee \circ)^*$

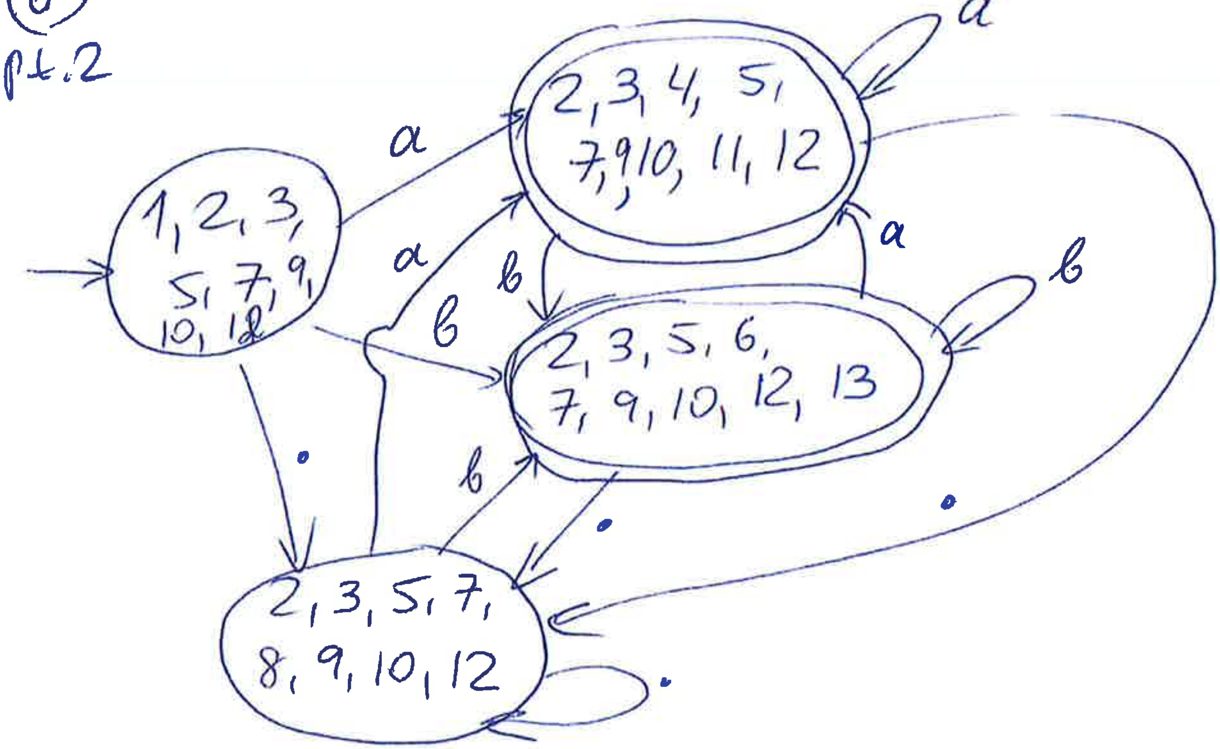



$(a \vee b \vee \circ)^* (a \vee b)$

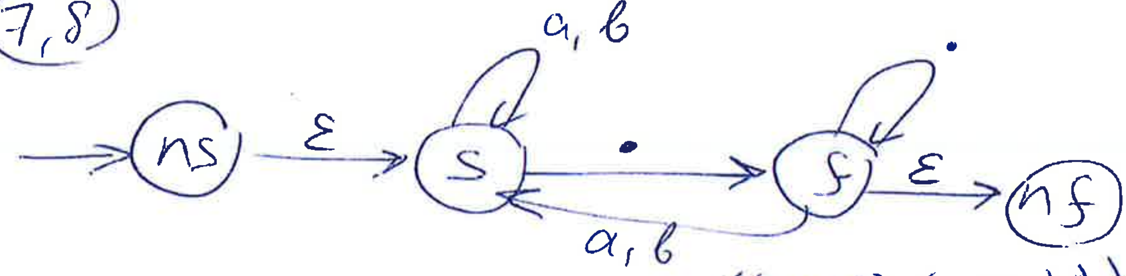


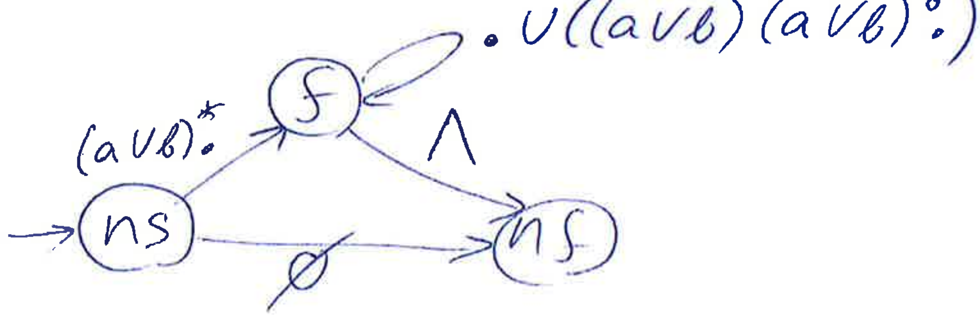
6

pt.2


1, 2, 3, 5, 7, 9, 10, 12
2, 3, 4, 5, 7, 9, 10, 11, 12
2, 3, 5, 6, 7, 9, 10, 12, 13
2, 3, 5, 7, 8, 9, 10, 12
a
a
a
a
b
b
b
b
.
.
.
.

7, 8




$$R'_{ns,f} = R_{ns,f} \cup (R_{ns,s} R_{s,s}^{*} R_{s,f}) =$$
$$\emptyset \cup (\Lambda \ (a \vee b)^{*} \cdot) = (a \vee b)^{*} \cdot$$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,s} R_{s,s}^{*} R_{s,nf}) =$$
$$\emptyset \cup (\Lambda (a \vee b)^{*} \emptyset) = \emptyset \cup \emptyset = \emptyset$$

$$R'_{f,nf} = R_{f,nf} \cup (R_{f,s} R_{s,s}^{*} R_{s,nf}) =$$
$$\Lambda \cup ((a \vee b)(a \vee b)^{*} \emptyset) = \Lambda \cup \emptyset = \Lambda$$

$$R'_{f,f} = R_{f,f} \cup (R_{f,s} R_{s,s}^{*} R_{s,f}) =$$
$$\cdot \cup ((a \vee b)(a \vee b)^{*} \cdot)$$


$$R'_{ns,nf} =$$
$$R_{ns,nf} \cup (R_{ns,f} R_{f,f}^{*} R_{f,nf}) =$$
$$\emptyset \cup ((a \vee b)^{*} \cdot (\cdot \cup ((a \vee b)(a \vee b)^{*} \cdot))^{*} \Lambda) =$$
$$\underline{(a \vee b)^{*} \cdot (\cdot \cup ((a \vee b)(a \vee b)^{*} \cdot))^{*}}$$

(9-10) Proof by contradiction. Let us assume that $L$ is regular. Then by pumping lemma
$\exists p \, \forall w_{\in L} \, (len(w) \geq p \rightarrow \exists x, y, z \, (w = xyz \,\&\, len(y) > 0 \,\&\, len(xy) \leq p \,\&\, \forall i \, (xy^i z \in L)))$
Let us take the word
$w = a^p b^p = \underbrace{a \ldots a}_{p \text{ times}} \underbrace{b \ldots b}_{p \text{ times}}$.
The length of this word is $p + p + 1 \geq p$, so by pumping lemma, this word can be represented as $xyz$ with $len(xy) \leq p$. The word starts with $xy$, and the length of $xy$ is $\leq p$, so $xy$ is among first $p$ symbols of $w$, and these symbols are all a's. So, $y$ has only a's.
Thus, when we go from $xyz$ to $xyyz$, we add a's but we do not add any b's. We started with word $xyz$ that has equal # of a's and b's. Thus, in $xyyz$, we have more a's than b's — the balance is disrupted, so $\underline{xyyz \notin L}$.
On the other hand, by pumping lemma for $i = 2$, $\underline{xyyz \in L}$. The contradiction shows that our assumption is false, and $L$ is not regular.