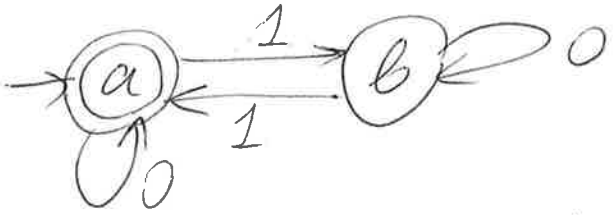
TEST 2, FALL 2025

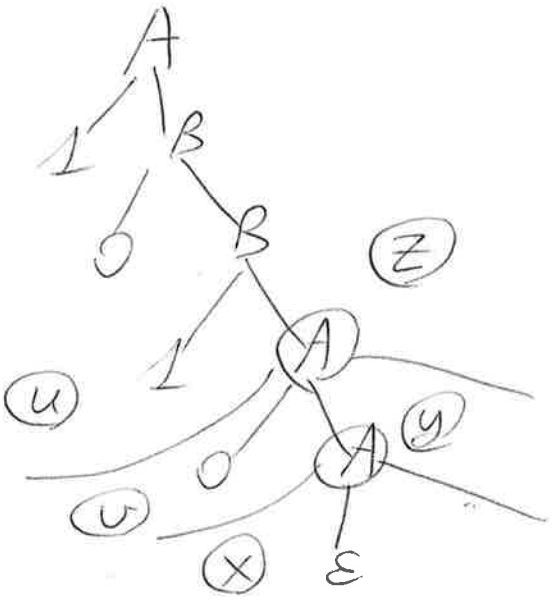


1

$A \to 1B \quad B \to 1A$

$A \to 0A \quad B \to 0B$

$A \to \varepsilon$

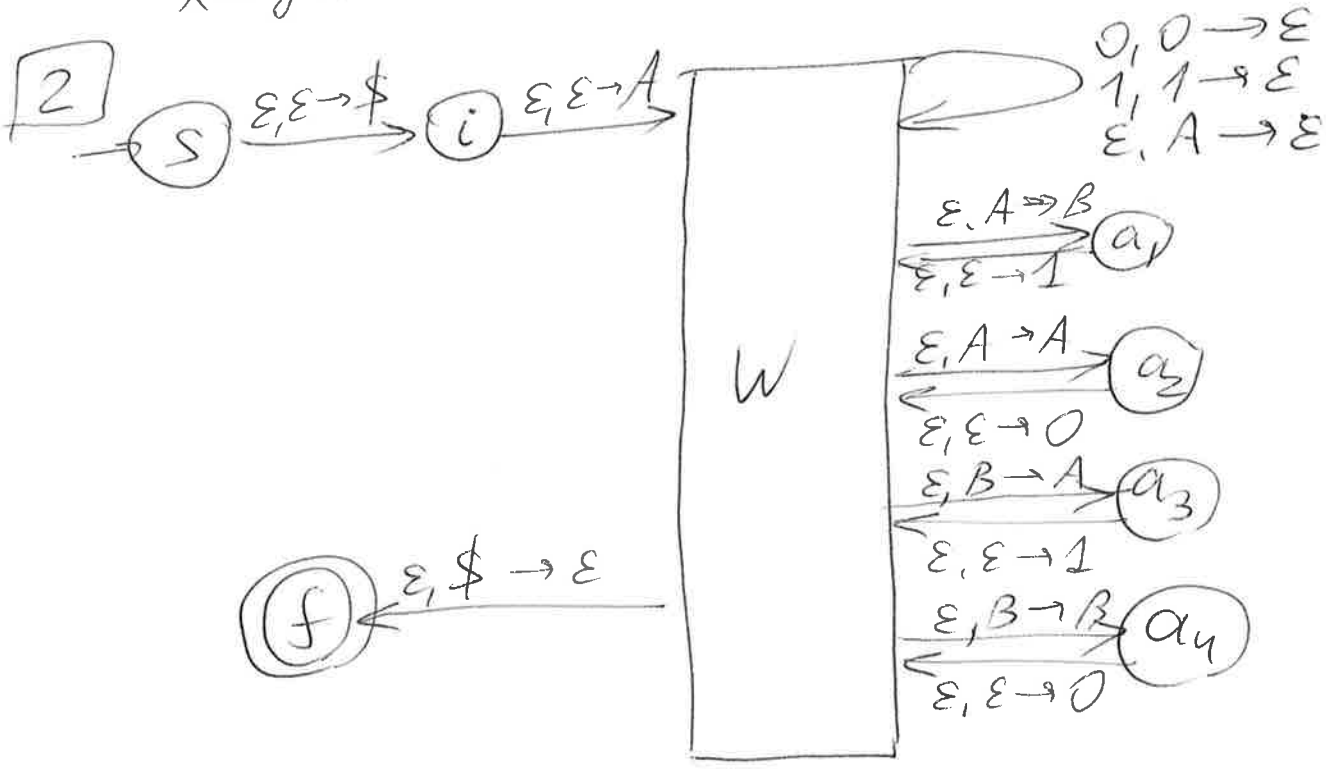


$u = 101$

$v = 0$

$x = y = z = \Lambda$

2



read			1			0			1			0				
state	s	i	w	a_1	w	w	a_4	w	w	a_3	w	w	a_2	w	w	w	f
stack	$	A	B	1	B	B	0	B	A	1	A	A	0	A	$		
		$	$	B	$	$	B	$	$	A	$	$	A	$			
				$			$			$			$				

3

1. ~~$A \to 1B$~~
2. ~~$A \to 0A$~~
3. ~~$B \to 1A$~~
4. ~~$B \to 0B$~~
5. ~~$A \to \varepsilon$~~

Precim_step

6. ~~$A_0 \to A$~~

Step 0

7. $A \to 0$
8. $B \to 1$
9. $A_0 \to \varepsilon$

Step 1

10. ~~$A_0 \to 1B$~~
11. ~~$A_0 \to 0A$~~
12. $A_0 \to 0$

Step 2

13. $V_0 \to 0$
14. $V_1 \to 1$
15. $A \to V_1 B$
16. $A \to V_0 A$
17. $B \to V_1 A$
18. $B \to V_0 B$
19. $A_0 \to V_1 B$
20. $A_0 \to V_0 A$

Step 3

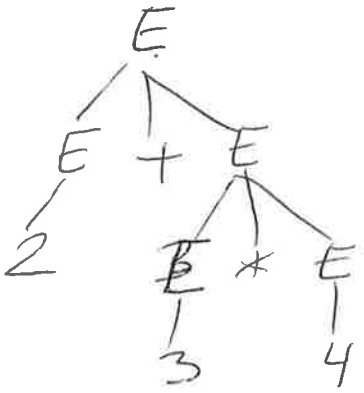
Not Needed

(4) $E \to 0$ $\quad E \to E+E$

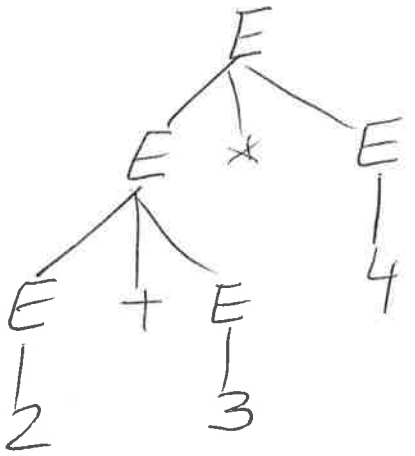
$\vdots$ $\quad E \to E*9$

$E \to 9$

Then 2+3*4 has two derivations:



← This is what we want



← this does not help in compiling.

S-6

Proof by contradiction.

Let us assume that L is CFG.

Then by pumping lemma

$\exists p \forall w \in L\ (len(w) \geq p \rightarrow \exists u,v,x,y,z / w = uvxyz$
$\& len(vy) > 0 \& len(vxy) \leq p \& \forall i (uv^i xy^i z \in L))$

Let us take $w = c^p b^p a^p = \underbrace{c \ldots c}_{p \text{ times}} \underbrace{b \ldots b}_{p \text{ times}} \underbrace{a \ldots a}_{p \text{ times}}$

Since $len(vxy) \leq p$, vxy cannot contain all 3 letters – otherwise it should contain at least one a, at least one c, and all p b's – to the total of $p+2 > p$. Thus, we have the following possibilities: (1) vxy is in a's, (2) vxy is in a's and b's, (3) vxy is in b's (4) vxy is in b's and c's, (5) vxy is in c's

In case 1, v and y only have a's so when we go from $uvxyz$ to $uvvxyyz$ we add a's but we don't add b's or c's. In $uvxyz$, we had equal number of a's, b's, and c's, now balance is $uvvxyyz$ is disrupted so $uvvxyyz \notin L$. But by pumping lemma for $i=2$, $uvvxyyz \in L$ Contradiction shows that this case is Not possible.

In Case 2, v and y only have a's and b's. So when we go from $uvxyz$ to $uvvxyyz$, we add a's and/or b's, but Not c's. So balance is disrupted, and $uvvxyyz \notin L$ – but by PL $uvxyyz \in L$.

Other cases are similar. In all cases we get contradiction, so L is Not CFG