CS 4365/CS 5354 Data Processing Under Security and Privacy Summer 2016, Test 1

Name:



1. Similarly to how we used Newton's method to design algorithms for computing square root and cubic root, design an algorithm for computing the logarithm $x = \ln(a)$ as a solution to the equation

$$e^{x} = a$$
.

$$F(x) = e^{x} - a = 0$$

$$F'(x) = e^{x}$$

$$\Delta x = -e^{x} - \alpha = \frac{\alpha - e^{x}}{e^{x}} = \frac{\alpha}{e^{x}} - \frac{e^{x}}{e^{x}} = \left(\frac{\alpha}{e^{x}} - 1\right)$$

$$X_{\text{next}} = X + \Delta X = X + (\% x - 1)$$

Assume
$$x^{(\omega)}=1$$
 $a=2$ $\ln(2)=0.693$

$$X_{\text{next}} = x + \Delta x = (1) + (2/e^{0} - 1) \approx 0.7357$$

$$x_{\text{next}} = 0.7357 + (2/60.7357 - 1) = 0.6940 \approx \ln(2) = 0.693$$

$$X_{\text{next}} = X + Dx = X + (2/e^{X} - 1)$$

2. Use the algorithm for computing 1/b that we had in class (and that is implemented in the computers) to perform the few first steps of computing the ratio 1/1.1.

$$2x^{(n)} - b(x^{n})^{2}$$

$$b = 1.1$$

$$x^{0} = 1$$

$$x^{(1)} = 2(1) - 1.1(1)^{2} = 0.9$$

$$x^{(2)} = 2(0.9) - 1.1(0.9)^{2} = 0.909$$

$$x^{(3)} = 2(0.909) - 1.1(0.909)^{2} = 0.90909$$

3-6.



- 3. Use numerical differentiation to compute the derivative of the function $x^2 x$ when x = 1.
- 4. Use linearization technique and your estimate for the derivative to estimate the range of this function when x is in the interval [0.9, 1.1].



5. Use naive interval computations to estimate the same range.

6. Use mean value form to estimate the same range.

3)
$$F(x+h)+F(x) = [(1.00)^{2} |.001]+[1^{2}-1]$$

 $h=0.001$ h = 1.001

4.)
$$\tilde{x} = 1$$

$$\Delta x = .1$$

$$\tilde{y} = 1^{2} - 1 = 1$$

$$\Delta = 2x \cdot \Delta x$$

$$= 2(1) \cdot 0.1$$

$$= 0.2$$

5)
$$x^2 = [0.9, 1.1] \cdot [0.9, 1.1] = [0.81, 1.21]$$

 $x^2 - x$
 $= [0.81, 1.21] - [0.9, 1.1] = [-0.29, 0.31]$

(e)
$$x=0.9$$
 $(0.9)^2-0.9=-0.09$
 $x=1.1$ $(1.1)^2-1.1=0.11$ $[-0.69,0.11]$

7. Use Newton's method to solve the following system of non-linear equations:

$$x_1 * x_2 = 3, x_1 + x_2 = 4.$$

Start with the first approximation $x_1 = 1$ and $x_2 = 2$. One iteration is good enough.

$$\begin{cases} X_{1}X_{2} = 3 & x_{1}^{(0)} = 1 \\ X_{1} + X_{2} = 4 & x_{2}^{(0)} = 2 \end{cases}$$

$$F_1(x_1,x_2) = x_1x_2 - 3$$

 $F_2(x_1,x_2) = x_1 + x_2 - 4$

$$\frac{\partial F_1}{\partial x_1} = X_2 = 2$$

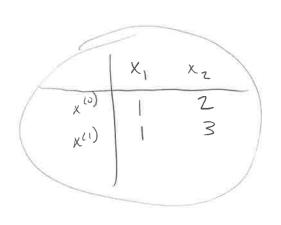
$$\frac{\partial F_2}{\partial x_2} = X_1 = 1$$

$$\frac{\partial F_2}{\partial x_1} = X_1 = 1$$

$$\frac{\partial F_2}{\partial x_2} = X_2 = 1$$

$$F_1(x^{(0)}) = -1$$

$$F_2(x^{(0)}) = -1$$



$$\Delta x_1 = 1 - \Delta x_2$$

$$2(1 - \Delta x_2) + \Delta x_2 = 1$$

$$2 - 2\Delta x_2 + \Delta x_2 = 1$$

$$-\Delta x_2 = 1$$

$$\Delta x_2 = 1$$

ZDX,+ DXz=+1

 $\Delta x_1 + \Delta x_2 = 1$

$$\Delta x_1 + 1 = 1$$

$$\Delta x_1 = 0$$

$$\Delta x_1 + 1 = 1$$
 $x_i^{(1)} = x + \Delta x_i$
 $x_i^{(1)} = 1 + 0 = 1$
 $\Delta x_1 = 0$
 $x_2^{(1)} = 2 + 1 = 3$

8. Find the point closest the origin on the line $x_1 - x_2 = 1$. In other words, find the values x_1 and x_2 for which the sum $(x_1)^2 + (x_2)^2$ attains the smallest possible value under the constraint $x_1 - x_2 = 1$.

$$x_{1}-x_{2}=|$$

$$x_{1}^{2}+x_{2}^{2}\rightarrow MiN$$

$$x_{1}^{2}+x_{2}^{2}+\lambda(x_{1}-x_{2}-1)\rightarrow min x_{1},x_{2}$$

$$\frac{\partial}{\partial x_{1}}=2x_{1}+\lambda=0$$

$$\frac{\partial}{\partial x_{2}}=2x_{2}-\lambda=0$$

$$2x_{1}+\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

$$2x_{2}-\lambda=0$$

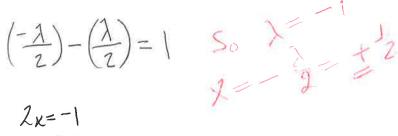
$$2x_1 + \lambda = 0$$

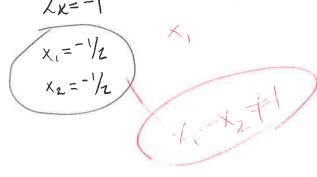
$$2x_2 - \lambda = 0$$

$$2x_1 = -\lambda$$

$$x_1 = -\frac{\lambda}{2}$$

$$x_2 = \frac{\lambda}{2}$$





9. Explain what is k-anonymity, and why it is important. If k increases, will we get more or less privacy protection? Explain your answer.

After all queries are taken we can narrow down to k different people per grouping. A box containing records of at least 2 different people.

With people grouped together it is much harder to single out a single person based off of the records it contains because if you look for one person's records you get information from at least 2 person's records.

If h increases you increase the number of People in each tox. Then you have more information per record per person on a single query of the data which makes more privacy protection. Which is alim to an inverse such that We, as h grave the charges of finding one person's records gets smaller because you get more information.