$\frac{90}{90} = \frac{100}{100}$

# CS 4365/CS 5354 Data Processing Under Security and Privacy Summer 2016, Test 1

Name: [redacted]

1. Similarly to how we used Newton's method to design algorithms for computing square root and cubic root, design an algorithm for computing the logarithm x = ln(a) as a solution to the equation

$$e^x = a.$$

10/10

$F(x) = e^x - a = 0$

$F'(x) = e^x$

$\Delta x = -\frac{e^x - a}{e^x} = \frac{a - e^x}{e^x} = \frac{a}{e^x} - \frac{e^x}{e^x} = \left(a/e^x - 1\right)$

$x_{next} = x + \Delta x = x + \left(a/e^x - 1\right)$

Assume $x^{(0)} = 1$ $a = 2$ $\ln(2) = 0.693$

$x_{next} = x + \Delta x = (1) + \left(2/e^{(1)} - 1\right) \approx 0.7357$

$x_{next} = 0.7357 + \left(2/e^{0.7357} - 1\right) = 0.6940 \approx \ln(2) = 0.693$

$x_{next} = x + \Delta x = x + \left(2/e^x - 1\right)$

2. Use the algorithm for computing 1/b that we had in class (and that is implemented in the computers) to perform the few first steps of computing the ratio 1 / 1.1.

$\frac{1}{1.1} = 0.90909$

10/10

$2x^{(k)} - b(x^{k})^2$

$b = 1.1$

$x^0 = 1$

$x^{(1)} = 2(1) - 1.1(1)^2 = 0.9$

$x^{(2)} = 2(0.9) - 1.1(0.9)^2 = 0.909$

$x^{(3)} = 2(0.909) - 1.1(0.909)^2 = 0.90909$

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3-6.

10/10

3. Use numerical differentiation to compute the derivative of the function $x^2 - x$ when $x = 1$.

9/10

4. Use linearization technique and your estimate for the derivative to estimate the range of this function when x is in the interval [0.9, 1.1].

5. Use naive interval computations to estimate the same range. 10/10

6. Use mean value form to estimate the same range. 10/10 + 3/10

3.) $\frac{F(x+h) - F(x)}{h} = \frac{[(1.001)^2 - 1.001] + [1^2 - 1]}{.001} = 1.001$

$h = 0.001$

4.) $\tilde{x} = 1$

$\Delta x = .1$

$= [1 - 0.2, 1 + 0.2]$

$\tilde{y} = 1^2 - 1 = 1$ $(2x - 1)$

$= [0.8, 1.2]$

$\Delta = 2x \cdot \Delta x$

$= 2(1) \cdot 0.1$

$= 0.2$

5.) $x^2 = [0.9, 1.1] \cdot [0.9, 1.1] = [0.81, 1.21]$

$x^2 - x$

$= [0.81, 1.21] - [0.9, 1.1] = [-0.29, 0.31]$

why?

6.) $x = 0.9$ $(0.9)^2 - 0.9 = -0.09$

$x = 1.1$ $(1.1)^2 - 1.1 = 0.11$

$= [-0.09, 0.11]$

7. Use Newton's method to solve the following system of non-linear equations:

$$x_1 * x_2 = 3, x_1 + x_2 = 4.$$

Start with the first approximation $x_1 = 1$ and $x_2 = 2$. One iteration is good enough.

10/10

$$\begin{cases} x_1 x_2 = 3 \\ x_1 + x_2 = 4 \end{cases} \qquad x_1^{(0)} = 1 \quad x_2^{(0)} = 2$$

$F_1(x_1, x_2) = x_1 x_2 - 3$

$F_2(x_1, x_2) = x_1 + x_2 - 4$

$\frac{\partial F_1}{\partial x_1} = x_2 = 2 \qquad \frac{\partial F_1}{\partial x_2} = x_1 = 1$

$\frac{\partial F_2}{\partial x_1} = x_1^0 = 1 \qquad \frac{\partial F_2}{\partial x_2} = x_2^0 = 1$

$F_1(x^{(0)}) = -1$

$F_2(x^{(0)}) = -1$

| | $x_1$ | $x_2$ |
|---|---|---|
| $x^{(0)}$ | 1 | 2 |
| $x^{(1)}$ | 1 | 3 |

$2\Delta x_1 + \Delta x_2 = +1$

$\Delta x_1 + \Delta x_2 = 1$

$\Delta x_1 = 1 - \Delta x_2$

$2(1 - \Delta x_2) + \Delta x_2 = 1$

$2 - 2\Delta x_2 + \Delta x_2 = 1$

$-\Delta x_2 = -1$

$\Delta x_2 = 1$

$\Delta x_1 + 1 = 1$

$\Delta x_1 = 0$

$x_i^{(1)} = x^{(0)} + \Delta x_i$

$x_1^{(1)} = 1 + 0 = 1$

$x_2^{(1)} = 2 + 1 = 3$

8/10

8. Find the point closest the origin on the line $x_1 - x_2 = 1$. In other words, find the values $x_1$ and $x_2$ for which the sum $(x_1)^2 + (x_2)^2$ attains the smallest possible value under the constraint $x_1 - x_2 = 1$.

$x_1 - x_2 = 1$

$x_1^2 + x_2^2 \to \min$

$x_1^2 + x_2^2 + \lambda(x_1 - x_2 - 1) \to \min_{x_1, x_2}$

$\frac{\partial}{\partial x_1} = 2x_1 + \lambda = 0$

$\frac{\partial}{\partial x_2} = 2x_2 - \lambda = 0$

$2x_1 + \lambda = 0$

$2x_1 = -\lambda$

$x_1 = \frac{-\lambda}{2}$

$2x_2 - \lambda = 0$

$2x_2 = \lambda$

$x_2 = \frac{\lambda}{2}$

$\left(\frac{-\lambda}{2}\right) - \left(\frac{\lambda}{2}\right) = 1$

So $\lambda = -1$

$x_1 = -\frac{\lambda}{2} = +\frac{1}{2}$

$2x = -1$

$x_1$

$x_1 = -1/2$

$x_2 = -1/2$

$x_1 - x_2 \neq 1$

9. Explain what is k-anonymity, and why it is important. If k increases, will we get more or less privacy protection? Explain your answer.

10/10

After all queries are taken we can narrow down to k different people per grouping. A box containing records of at least 2 different people. With people grouped together it is much harder to single out a single person based off of the records it contains because if you look for one person's records you get information from at least 2 person's records.

If k increases you increase the number of people in each box. Then you have more information per record per person on a single query of the data which makes more privacy protection. Which is akin to an inverse such that $1/k$, as k grows the chances of finding one person's records gets smaller because you get more information.