$\frac{92}{90} = \frac{102}{100}$

CS 4365/CS 5354 Data Processing Under Security and Privacy
Summer 2016, Test 1

Name: [redacted]

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1. Similarly to how we used Newton's method to design algorithms for computing square root and cubic root, design an algorithm for computing the logarithm x = ln(a) as a solution to the equation

$$e^x = a.$$

$F(x) = e^x - a = 0$

$F'(x) = e^x$

$\Delta x = \frac{-F(x^{(k)})}{F'(x^{(k)})}$

$= \frac{-e^x - a}{e^x} = \frac{a}{e^x} - 1$

$x^{(0)} = 1$ and $x^{(next)} = x^{prev} + \Delta x$

$\boxed{x^{next} = x^{prev} + \frac{a}{e^{x^{prev}}} - 1}$ with the more iterations, the more accurate.

i.e. computing $\ln(3)$

$\Delta x = \frac{3}{e^1} - 1 = .104$

$x^{(1)} = 1 + .104 = 1.104$

$x^{(2)} = 1.104 + \frac{3}{e^{1.104}} - 1 = 1.0986$ ✓

actual $= 1.0986$ ✓

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2. Use the algorithm for computing 1/b that we had in class (and that is implemented in the computers) to perform the few first steps of computing the ratio 1 / 1.1.

$1/b \approx a$ (a is the solution we're seeking)

$a \cdot b \simeq 1$

$\Delta a = a(1-ab)$

$a^{(k+1)} = a^{(k)} + \Delta a$ ($a^{(k)}$ is approximation)

$a^{(0)} = 1$

$b = 1.1$

$a^{(1)} = 1 + 1(1-1.1) = .9$

$a^{(2)} = .9 + .9(1-(1.1)(.9))$

$= \underline{0.909}$

actual $= 0.\overline{909}$, so close enough ✓

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3-6.

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3. Use numerical differentiation to compute the derivative of the function $x^2 - x$ when $x = 1$.

4. Use linearization technique and your estimate for the derivative to estimate the range of this function when x is in the interval [0.9, 1.1]. 10/10

5. Use naive interval computations to estimate the same range. 10/10

6. Use mean value form to estimate the same range. 10+3/10

3) $f(x) = x^2 - x$

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$, where h is small, in this case $h = 0.01$

$x = 1$

$$f'(x) = \frac{(1.01^2 - 1.01) - (1^2 - 1)}{.01} = \underline{1.01}$$

check

$f'(x) = 2x - 1 = 2(1.01) - 1 = 1.02$ ✓

4) $f(x) = x^2 - x$, $x \in [0.9, 1.1]$

$\tilde{x} = \frac{.9 + 1.1}{2} = 1$

$\Delta_1 = \frac{1.1 - .9}{2} = .1$

$\tilde{y} = 1^2 - 1 = 0$

$\Delta = \sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right| \Delta_i$

$= |2x - 1| \cdot (.1)$

$= |2 - 1| \cdot .1 = \underline{.1}$

range $= [0 - .1, 0 + .1] \Rightarrow \underline{[-.1, .1]}$

5) Naive

$f(x) = x^2 - x \quad , \quad x \in [.9, 1.1]$

$$f(x) = [.9, 1.1]^2 - [.9, 1.1]$$
$$= [.81, 1.21] - [.9, 1.1]$$
$$= [-0.29, 0.31] \Rightarrow \text{range}$$

6) Mean value form

$f(x) = x^2 - x \quad , \quad x \in [.9, 1.1]$

$f'(x) = 2x \Rightarrow$ increasing so not

can just use regular

$\underline{x} = .9^2 - .9 = .81 - .9 = -.09$

$\overline{x} = 1.1^2 - 1.1 = 1.21 - 1.1 = .11$

So range is $[-0.09, .11]$

7. Use Newton's method to solve the following system of non-linear equations:

$$x_1 * x_2 = 3, x_1 + x_2 = 4.$$

Start with the first approximation $x_1 = 1$ and $x_2 = 2$. One iteration is good enough.

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$F_1: x_1 * x_2 - 3$

$F_2: x_1 + x_2 - 4$

$\frac{\partial F_1}{\partial x_1} = x_2 = 2$ $\frac{\partial F_1}{\partial x_2} = x_1 = 1$

$\frac{\partial f_2}{\partial x_1} = 1$ $\frac{\partial f_2}{\partial x_2} = 1$

$x_1^{(0)} = 1, x_2^{(0)} = 2$

$F_1(x^{(0)}) = -1$, $F_2(x^{(0)}) = -1$

$\sum \frac{\partial F_i}{\partial x_i} \Delta x_i = -f_i(x^{(k)})$

$\begin{cases} 2\Delta x_1 + \Delta x_2 = 1 & (1) \\ \Delta x_1 + \Delta x_2 = 1 & (2) \end{cases}$

"(1)" $\Delta x_2 = 1 - 2\Delta x_1$

(2) $\Delta x_1 + 1 - 2\Delta x_1 = 1$

$\underline{\Delta x_1 = 0}$

(1) $\underline{\Delta x_2 = 1}$

$x_i^{(k+1)} = x_i^{(k)} + \Delta x_i$

$\underline{x_1^{(1)} = 1 + 0 = 1}$

$\underline{x_2^{(1)} = 2 + 1 = 3}$

Quick check

$(1) \cdot (3) = 3$ ✓ $(1) + (3) = 4$ ✓

8. Find the point closest the origin on the line $x_1 - x_2 = 1$. In other words, find the values x_1 and x_2 for which the sum $(x_1)^2 + (x_2)^2$ attains the smallest possible value under the constraint $x_1 - x_2 = 1$.

$\frac{9}{10}$

$f = (x_1)^2 + (x_2)^2 \rightarrow \min$

$g(x) = x_1 - x_2 - 1$

$(x_1)^2 + (x_2)^2 + \lambda(x_1 - x_2 - 1) \rightarrow \min_{x_1, x_2}$

$\partial/\partial x_1 : 2x_1 + \lambda$

$\partial/\partial x_2 : 2x_2 - \lambda$

$x_1 = \frac{-\lambda}{2}, \quad x_2 = \frac{\lambda}{2}$

$X_1 = -X_2$ for all parameters λ

$g(x) = X - (-x) = 1$

$2x = 1$

$x = 1/2$

$f(x) = (1/2)^2 + (-1/2)^2 \rightarrow \min$

$= 1/2$

Point closest to origin is $(1/2, 1/2)$

$x_1 - x_2 \neq 1$

9. Explain what is k-anonymity, and why it is important. If k increases, will we get more or less privacy protection? Explain your answer.

10/10

K-anonymity is the when after all the queries are done, the people are narrowed down to are ≥ k different people. This is important because it increases the privacy of the users by following the ≥ k rule.

If k increases, there will be more privacy protection as there will be more people to chose from, and thus be harder to pinpoint the exact person per query.