Final Exam for Uncertainty in AI class, Fall 2025

1. Suppose that we know the measurement results $\widetilde{x}_1, \ldots, \widetilde{x}_n$, the data processing algorithm $y = f(x_1, \ldots, x_n)$, and the standard deviations $\sigma_1, \ldots, \sigma_n$.

a) Describe how to best estimate the standard deviation σ of the data processing result y when n is small (provide the formula) and how to estimate it when n is large (just explain the idea).

b) What will be the standard deviation when $y = x_1 - x_2$, $\sigma_1 = 0.3$, and $\sigma_2 = 0.4$?

2. Suppose that we know the measurement results $\widetilde{x}_1, \ldots, \widetilde{x}_n$, the data processing algorithm $y = f(x_1, \ldots, x_n)$, and the upper bounds $\Delta_1, \ldots, \Delta_n$ on the absolute values of the measurement errors.

a) Describe how to best estimate the upper bound Δ of the absolute value of the approximation error of the measurement result y when n is small (provide the formula) and how to estimate it when n is large (just explain the idea).

b) Describe two situations in which we only know such upper bounds and briefly explain why in these situations, we do not know the probabilities.

c) What will be the value Δ when $y = x_1 - x_2$, $\Delta_1 = 0.3$, and $\Delta_2 = 0.4$?

3. Suppose that we use bisection to compute $\sqrt{3}$, i.e., the solution to the equation $f(x) = 0$ when $f(x) = x^2 - 3$. We know that $f(1) = 1^2 - 3 = 1 - 3 = -2 < 0$ and that $f(3) = 3^2 - 3 = 9 - 3 = 6 > 0$, so we know that the solution is somewhere on the interval $[1, 3]$. When we follow bisetion method, what would we do next, and what will be the resulting new narrower interval?

4. Suppose that $y = f(x_1, x_2) = x_1 - x_2$. Suppose that with confidence 0.5, experts believe that the actual value of x_1 is in the interval $[1, 2]$, and that actual value of x_2 is in the interval $[2, 3]$. Describe the corresponding alpha-cut for y.

Turn over, please

5.

a) If 7 experts out of 10 believe that the statement A is true, what is the resulting degree of confidence?

b) Suppose that our degree of confidence in a statement A is 0.8, in a statement B is 0.7 Suppose that we use min as "and" and max as "or". What is our estimate for the degree of confidence in a composite statement $A \vee \neg B$?

6.

a) How many binary questions do we need to ask a user to get his/her utility of a given alternative with accuracy 1%?

b) Assuming that utility is proportional to the square root of money amount, would a person prefer $9 without any condition or $100 with probability 0.1?

7. Suppose that we have two alternatives, with gains $[2, 5]$ and $[3, 4]$.

a) Which of them are possibly optimal? definitely optimal?

b) Which of the alternatives should we choose if Hurwicz coefficient α_H is 0.6?

8–10.

8. Prove that for each final invariant optimality criterion, the optimal alternative is itself optimal.

9. Use this result to explain why ReLU is the optimal activation function.

10. Use the result from Problem 8 to explain why we should use $1 - x$ for negation, $a \cdot b$ for "and", and $a + b - a \cdot b$ for "or".