Solution to Homework 8

Question. If a value is -1 with probability between 0.2 and 0.4, 0 with probability between 0.4 and 0.6, and 1 with the remaining probability, what is the interval value $F(-0.5)$ of the corresponding p-box? what is the value of $F(0.5)$?

Answer. Out of the three possible cases $x = -1$, $x = 0$, and $x = 1$, the only case when $x \leq -0.5$ is when $x = -1$. The probability of this case is 0.3, so:

$$F(-0.5) = \text{Prob}(x \leq -0.5) = [0.2, 0.4].$$

We have two cases when $x \leq 0.5$: the cases of $x = -1$ and $x = 0$. Thus, the probability that $x \leq 0.5$ is equal to the sum of these two cases:

$$F(0.5) = \text{Prob}(x \leq 0.5) = \text{Prob}(x = -1) + \text{Prob}(x = 0) = [0.2, 0.4] + [0.4, 0.6].$$

In general:

- the smallest possible value of the sum of the two numbers known with uncertainty is when both numbers are the smallest possible, and

- the largest possible value of the sum of the two numbers known with uncertainty is when both numbers are the largest possible.

So, to find the range of possible values of the above sum of two probabilities, we need to add the lower endpoints of these two intervals and add the upper endpoints:

$$F(0.5) = [0.2, 0.4] + [0.4, 0.6] = [0.2 + 0.4, 0.4 + 0.6] = [0.6, 1].$$